SCIENCE FOR A BETTER LIFE:
DEVELOPING REGIONAL SCIENTIFIC PROGRAMS IN PRIORITY AREAS
FOR LATIN AMERICA AND THE CARIBBEAN

VOLUME 4

MATHEMATICS EDUCATION IN LATIN AMERICA AND THE CARIBBEAN: A REALITY TO BE TRANSFORMED

CARLOS BOSCH • LILLIAM ÁLVAREZ DÍAZ • RAFAEL CORREA • SUELY DRUCK • RAYMOND McEACHIN

ICSU
International Council for Science
Regional Office for
Latin America and the Caribbean

CONACYT
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FOREWORD

Founded in 1931, the International Council for Science (ICSU) is a non-governmental organization that plans and coordinates interdisciplinary research to address major issues of relevance to both science and society. Over the years the geographical breadth of ICSU activities has changed. Increasingly a major emphasis for ICSU has been the development of scientific capacity in developing countries and the integration of these scientists in international research initiatives.

The creation of three ICSU Regional Offices, established in Africa, Asia and the Pacific, and Latin America and the Caribbean also marks a fundamental change in ICSU structure, the aim of which is two-fold. First, it should enhance the participation of scientists and regional organizations from developing countries in the programs and activities of the ICSU community. Secondly, it will allow ICSU to play a more active role in strengthening science within the context of regional priorities through scientific collaboration.

Especially in regard to Latin America and the Caribbean, this is an important step in bridging the 'islands of competence' that exist in every country and that together will be able to advance significantly the scientific research agenda in the region. The first step towards the establishment of a Regional Office was the appointment in 2006 of the Regional Committee for Latin America and the Caribbean, composed of renowned scientists of the region.

The Regional Office for Latin America and the Caribbean was the third to be established and was inaugurated in April 2007. It is hosted by the Brazilian Academy of Sciences, in Rio de Janeiro, Brazil, and supported by the Brazilian Ministry of Science and Technology, ICSU, and CONACYT Mexico. From October 2010 it will be hosted by the Mexican Academy of Science, with the support of CONACYT Mexico.

Based on the ICSU Strategic Plan 2006-2011, the Regional Committee has selected four priority areas to be developed:

- Mathematics Education;
- Biodiversity: knowledge, preservation and utilization of biodiversity of all countries of the Latin American and Caribbean region, and to ensure that the scientific community of the smaller countries of the region are fully integrated in DIVERSITAS;
- Natural Hazards and Disasters: prevention and mitigation of risks especially of hydrometeorologic origin with special attention to the necessary social science research;
- Sustainable Energy: assessment of the existing capacities in the LAC region and the social impact of the use and development of new energy resources.

Four Scientific Planning Groups were appointed to develop proposals that reviewed the current status of the priority area in the region and to formulate a set of detailed objectives and targeted areas of research to be developed in the next few years.

Engaging highly qualified scientists from Latin America and the Caribbean, the Scientific Planning Groups did outstanding work within a restricted time limit. We thank each and every one of the participants for their enthusiasm and dedication.

This document is the final report from the Scientific Planning Group on Mathematics Education, which is being submitted to the scientific community in the expectation of effectively influencing mathematics education in the years to come.

José Antonio de la Peña
Chair
Regional Committee
for Latin America and the Caribbean

Alice Abreu
Director
Regional Office
for Latin America and the Caribbean

EXECUTIVE SUMMARY

All over the world, there is growing concern about mathematics and science education. From the evidence gathered during the last years, it is clear that we are not doing this task properly. And that means a big problem for all Latin American and Caribbean countries. It is also clear that our children are falling behind. Mathematics and science are primary sources of lifelong learning and progress for our civilization. We have to do something, especially in the countries that were the land of brilliant civilizations like the Mayans and the Incas, who made important scientific and mathematical contributions.

In Latin America and the Caribbean, children are not being taught at the level they will need to live their lives and work productively. Proof of this can be found in the results of international evaluations like the Trends in the International Mathematics and Science Study (TIMSS), which provide reliable and timely data on mathematics and science achievement in other countries.

Chile was the only country from our region that participated in the study, and the results were poor, especially if we compare them with Morocco, a country with a per capita GDP almost 9 times smaller than Chile's.

There are other international evaluations, such as the Program for International Student Assessment (PISA) from the OECD, which evaluates students' skills and knowledge as they approach the end of their compulsory education. This is not a conventional school test. Rather than examining how much the students have learned from their school curriculum, it looks at how well prepared they are for life

beyond school. The PISA evaluates reading and mathematics abilities. In many countries, low mathematics results correlate with low language results. These are the reading and mathematics results for some countries found in the OECD's 2006 PISA database:

Selected countries	Reading	Mathematics
Korea	556 (maximum)	547
Finland	547	548 (maximum)
Fited Kingdom	495	495 (average)
Slovenia	494 (average)	504
Spain	461	480
Chile	442	411
Uruguay	413	427
Mexico	410	406
Brazil	393	370
Colombia	385	370
Argentina	374	381
Kyrgyzstan	285 (minimun)	311 (minimun)

Most LAC countries provide education for almost all their young people. Regrettably, education quality is too often poor. Long-term solutions are urgently needed. The region needs better training for its mathematics teachers. In few words, the region needs: BETTER TEACHERS.

How can this be achieved? In many countries from the region, the responsibility for educating teachers tends to be scattered among different agencies, including Ministries of Education, teacher training colleges and programs, university-level departments of education, and some other institutions. Except for a handful of educators trained by departments of mathematics, the role of mathematicians

in the training process has been limited or even nonexistent throughout most of LAC. This suggests that mathematicians can play a greater role in teacher education and training, provided that care is taken in their approach. Certainly, if departments of mathematics throughout the region simply start trying to "improve" teacher's training without a proper dialogue them and their full cooperation, they risk fragmenting even further the training process.

However, if departments of mathematics engage with the various agencies in their respective countries, and achieve a real dialogue with the teachers and summon their cooperation, there is a good chance that they can help unify and improve the process of training mathematics teachers for their national education system.

This would allow the use of a mostly untapped resource to help improve the teaching of mathematics throughout the region. The aim of this document is to explore all possible actions that point in that direction.

In order to achieve this, the following detailed objectives are recommended:

- Reinforcing the link between professional mathematicians and school teachers.
- Developing suitable data for performance evaluation.
- Defining sets of minimum standards for mathematics teachers at the elementary and intermediate school levels throughout the region.
- Using information and communication technologies (ICT) to make more resources available for teachers and students.
- Finding strategies to improve simultaneously mathematical and language literacy.
- Continuing the involvement of mathematics education faculties in supporting mathematics teachers on subjects such as pedagogy and didactics.

How can national education systems get the right people to pursue mathematics teaching careers? How can we help them develop into more effective instructors and raise the standards of every student? The following specific actions are proposed.

- Linking mathematicians with mathematics education. An important action will be completing the assessment of the existing good practices, and sharing and expanding those experiences to other countries in the region.
- Developing suitable data. Many LAC countries perform national evaluations for several age groups. But to allow for data comparisons between LAC countries, as well as comparisons against nations from other regions, all countries should be encouraged to participate in standardized international evaluations like PISA or TIMSS.
- Initial teacher training. An especially important contribution would be setting regional standards for initial teacher training.
- In-service training for teachers. The transition from the initial training notion to that of continuing education is the landmark of modern pedagogy. This document proposes several ways to assist in-service teachers by linking them with professional mathematicians.
- School-based national Olympics. Many LAC countries have had good experiences with Mathematics Olympics. This proposal emphasizes the need to increase the quality of mathematics teaching by competitions that connect professional mathematicians with students and teachers right in their schools. As other Olympic games, school-based national Olympics are competitions that feature exams at several stages, and the awards go to the top performers. However, they have additional characteristics that guarantee the involvement of schools, local mathematics school teachers, and national-level professional mathematicians.

Mathematics education can no longer be seen as something that is only appropriate for a handful of students, who will pursue scientific-based careers in the future, or for especially gifted or motivated students, or exclusively for males. Presently, mathematics education is understood as a right for all students, and as a specific type of training for life. More than half of LAC's students are below PISA Level 1, which means that most of them cannot identify mathematical information, nor carry out routine procedures, according to direct instructions in explicit situations.

We need to act urgently. Although some countries have had successful experiences when trying to reverse the depicted situation, that is not enough, and this is not happening in all countries. It would be better if we could get all LAC countries to act together, since their problems are similar. This report identifies some actions that can be multiplied or introduced across the region.

In all of them, professional mathematicians are required to help teachers achieve the necessary mathematical knowledge, and become more effective instructors, in order to raise the standards of all students.

This is URGENT; and something must be done.

SCIENTIFIC PLANNING GROUP AND REGIONAL COMMITTEE

Scientific Planning Group on Mathematics Education

Carlos Bosch *Chair* (Instituto Tecnológico Autónomo de México, Mexico)
Lilliam Álvarez Díaz (Academia de Ciencias de Cuba, Cuba)
Rafael Correa (Centro de Modelamiento Matemático, Universidad de Chile, Chile)
Suely Druck (Universidade Federal Fluminense, Brazil)
Raymond McEachin (University of the West Indies, Jamaica)

Juan A. Asenjo *Liaison RCLAC* (Chile)

ICSU Regional Committee for Latin America and the Caribbean (2006-2012)

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Ex officio

Jerson Lima Silva (ABC)
Sergio Pastrana (ICSU Executive Board)
Patricia Ocampo-Thomason (ICSU Secretariat)
Alice Abreu (ICSU Regional Office)

CONTENT

ACKNOWLEDGMENTS

Since its beginnings in 2007, the preparation of this report has involved the support of a large number of persons and institutions. First of all, we want to thank the ICSU Regional Committee for Latin America and the Caribbean for appointing us as the Scientific Planning Group (SPG) on Mathematics Education.

In all of our meetings, we had the presence of Alice Abreu, director of the Regional Office for Latin America and the Caribbean, which was a privilege. Thanks to her, while working hard, we also enjoyed moments of conviviality. She was the keystone that bound our group. Thank you very much Alice.

We were also very lucky that the Regional Committee for Latin America and the Caribbean designated Professor Juan Asenjo as liaison for this SPG. We greatly appreciate all his cheerfulness and expertise, as well as his time and effort in contributing to the report.

We held three lengthy meetings, which took place in Mexico City, Santiago de Chile, and Rio de Janeiro. Each one of them was kindly hosted and supported by the Academia Mexicana de Ciencias, and the Consejo Nacional de Ciencias y Tecnología (CONACYT) in Mexico; the Centro de Modelamiento Matemático (CMM) of the University of Chile; and the Academia Brasileria de Ciências and the Instituto Nacional de Matemática Pura e Aplicada (IMPA) in Brazil. Without the valuable support from these institutions, our work would have been harder, so we feel deeply grateful to them.

We also want to thank all the ICSU team members, but especially Sybelle de Jong, who was always there to help us find flight reservations, transportation, and other small details that, if not taken well care of, can become gigantic problems.

Finally, we want to thank the arbitrators for their suggestions, which gave us the opportunity to produce a better document.

Carlos Bosch
Chair of the SPG on Mathematics Education

1. INTRODUCTION

All over the world, there is growing concern about mathematics and science education. Mathematics is a key ability in the teaching of virtually every discipline. However, in Latin America and the Caribbean (LAC), most students lack the ability to solve problems even with basic algorithms, and are unable to perform direct reasoning. This is a tremendous cultural disadvantage, and particularly so in a knowledge-based society that requires the crucial and important abilities given by mathematics. Mathematics requires logic, precise, rigorous, abstract, and formal structured thinking. Using mathematical abilities makes also possible to decide and recognize the essentials, the appreciation of intellectual creation and the potential of science. All school topics must contribute to nurture and develop student's intelligence, emotional abilities, and personality, and mathematics represents a key element in this matter. In our daily life, mathematics is important to understand and analyze the flow of information that we receive from the media, and it encompasses many other human endeavors, since most of these include mathematically structured models. In our modern society, many activities are based on mathematics. The applications of this discipline range from basic principles, environment and finances to energy allocation, and from information and communication technologies to accounting statements. Clearly, if we want to be a relevant player in the globalized world that looms in front of us, it will be essential to have young people well prepared in mathematics.

In fact, the importance of education is growing all over the world. The relationship between education and development, as well as the role of education in social inclusion and civil participation is increasingly evident. Education is being appreciated as never before, both as a human right and as a factor for development and collective achievement of higher levels of civilization. Within such framework, mathematics and science education is of primary importance due to our increasingly interdependent global economy, as well as to the globalized labor market and the technological developments that characterize our era and the near future. This is recognized by numerous documents, such as the report "Before It's Too Late: Report to the Nation from The National Commission on Mathematics and Science Teaching for the 21st Century".

Now, mathematics education is considered as the basic right that all students have to receive a specific type of training for life. Following the definition used by the OECD's Program for International Student Assessment (PISA): "Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen."

According to this new awareness, countries are participating increasingly in large scale national and international assessments [such as PISA and Trends in the International Mathematics and Science Study (TIMSS)] and making considerable efforts to improve the performance of their educational systems. However, LAC countries have not achieved the expected results in mathematics education.

Research on mathematics education has clearly shown the difficulty of teaching mathematics, even at the elementary school level. This research reveals that teaching mathematics is a highly demanding mathematical task (Ball 2002).

In this context, the participation of professional mathematicians in mathemat-

ics training projects would be highly appreciated. Also, their participation in institutional programs that help develop this sector, and in the corresponding public policy discussions, would be very important.

To address these issues, the ICSU Regional Office for LAC convened a Scientific Planning Group (SPG) whose task was to analyze the current situation and to develop a proposal for improving mathematics education in the region. This document is the final report from the SPG, and its core proposal is the involvement of the professional mathematicians from the region to improve and strengthen the national mathematical educational systems. It also contains policy recommendations, and specific actions to achieve these goals.

2. MATHEMATICS EDUCATION IN LAC: OUR REALITY

Latin American and Caribbean countries are very different in some aspects, while at the same time, very similar in others. For instance, population and per capita gross domestic product (GDP) separate the region's countries into very different groups. Some countries, like Brazil and Mexico, are densely populated (176 million and 105 million inhabitants, respectively), while others, like Antigua and Barbuda, and Saint Kitts and Nevis, have quite small populations (77 000 and 48,000 inhabitants, respectively). On the other hand, the region's per capita purchasing power parity (ppp) ranges from 3 430 USD in Honduras and 3 674 USD in Nicaragua, to 14 280 USD in Argentina and 14 603 USD in Trinidad and Tobago. And this is even more dramatic when compared to countries like Canada, whose per capita ppp reaches 33 345 USD (Table 1).

Latin America and the Caribbean is comprised by 35 countries, and only three main languages —Spanish, Portuguese, and English— are spoken, so almost everybody is able to communicate in this region of the world. Thus, language is a regional strength. Among the similarities, literacy for the 15-24 year age group, which is estimated at nearly 90%, is similar in all LAC countries; on the contrary, there are great differences in the bracket corresponding to older people.

All over the world, there is growing concern about mathematics and science education. From the evidence gathered during the last years, it is clear that we are not doing this task properly. And that is a big problem for LAC countries. It is

Table 1. Social indicators for selected countries

	Total population (thousands)	Annual population growth (%)	Life expect. at birth	Poverty (% of pop. with less than 2 USD/day)	GDP (per capita ppp in USD)
Antigua & Barbuda	–	1.7	75	–	12 134
Argentina	38 747	1.0	75	17	14 280
Barbados					
Bolivia					
Belize	276	3.2	72	–	7 109
Venezuela	26 726	1.7	74	40	6 632
Brazil	186 831	1.3	71	21	8 402
Chile	16 295	1.1	78	6	12 027
Colombia	44 946	1.5	73	18	7 304
Costa Rica	4 327	1.7	79	10	10 180
Cuba	11 260	0.2	77	–	–
Dominican Republic	9 470	1.4	68	16	8 217
Ecuador	13 061	1.4	75	–	4 341
El Salvador	6 668	1.7	71	41	5 255
Guatemala	12 710	2.4	68	32	4 568
Guyana	739	0.1	64	6	4 508
Haiti					
Honduras	6 834	2.2	69	36	3 430
Jamaica	2 682	0.5	71	14	4 291
Mexico	104 266	1.0	75	12	10 751
Nicaragua	5 463	0.5	70	80	3 674
Panama	3 232	1.8	75	18	7 605
Paraguay	5 904	1.9	71	30	4 642
Peru	27 274	1.5	71	31	6 039
Saint Kitts and Nevis	43	2.1	71	–	12 917
Suriname	452	0.6	70	–	7 722
Trinidad and Tobago	1 324	0.3	70	–	14 603
Uruguay	3 326	0.7	76	6	9 962
Canada	32 271	1.0	80	0	33 375

Source: http://portal.unesco.org/ci/en/ev.php-URL_ID=1352&URL_DO=DO_TOPIC&URL_SECTION=201.html

also clear that our children are falling behind. Mathematics and science are primary sources of lifelong learning and progress for our civilization. Something has to be done to improve this situation, especially if we consider that some of these countries were the lands of brilliant civilizations like the Mayans and the Incas, which made important scientific and mathematical contributions. In LAC, children are not being taught at the level they will need to live their lives and work productively. Proof of this can be found in the results of international evaluations like the Trends in the International Mathematics and Science Study (TIMSS), which provide reliable and timely data on mathematics and science achievement in other countries. The TIMSS has collected data for 1995, 1999, 2003, and 2007. The 2007 results will be available by the end of 2008. The 2003 TIMSS was the third and most recently completed round of this ambitious series of international assessments, carried out throughout the world to measure trends in mathematics and science learning at the fourth and eighth grades. A summary of the overall distribution of mathematical achievement is shown in Table 2.

Chile was the only country from our region that participated in the TIMSS, and the results were poor, especially if we compare them with Morocco, a country with a per capita GDP nearly 9 times smaller than Chile's.

There are other international evaluations, such as the Program for International Student Assessment (PISA) from the OECD, a survey which measures the students' skills and knowledge as they approach the end of their compulsory education. This is not a conventional school test. Rather than examining how much the students have learned from their school curriculum, it looks at how well prepared they are for life beyond school. All the countries that participated in the 2006 PISA are members of the OECD, except those marked with an asterisk (*):

Table 2. Trends in International Mathematics and Science Study (TIMSS, 2003), selected countries

Countries	Overall average score on the scale	Years of formal schooling	Average age
Singapore	605	8	14.3
South Korea	589	8	14.6
Japan	570	8	14.4
Belgium	537	8	14.1
The Netherlands	536	8	14.3
Estonia	531	8	15.2
Hungary	529	8	14.5
Malaysia	508	8	14.3
Russian Federation	508	8	14.2
Slovak Republic	508	8	14.3
Australia	505	8	13.9
United States	504	8	14.2
Lituania	502	8	14.9
Sweden	499	8	14.9
New Zealand	494	8.5–9.5	14.1
Italy	484	8	13.9
International Average	467	8	14.5
Norway	461	7	13.8
Tunisia	410	8	14.8
Egypt	406	8	14.4
Bahrain	401	8	14.1
Chile	387	8	14.2
Morocco	387	8	15.2
Philippines	378	8	14.8
Botswana	366	8	15.1
Saudi Arabia	332	8	14.1
Ghana	276	8	15.5
South Africa	264	8	15.1

Source: timss.bc.edu/timss2003.html

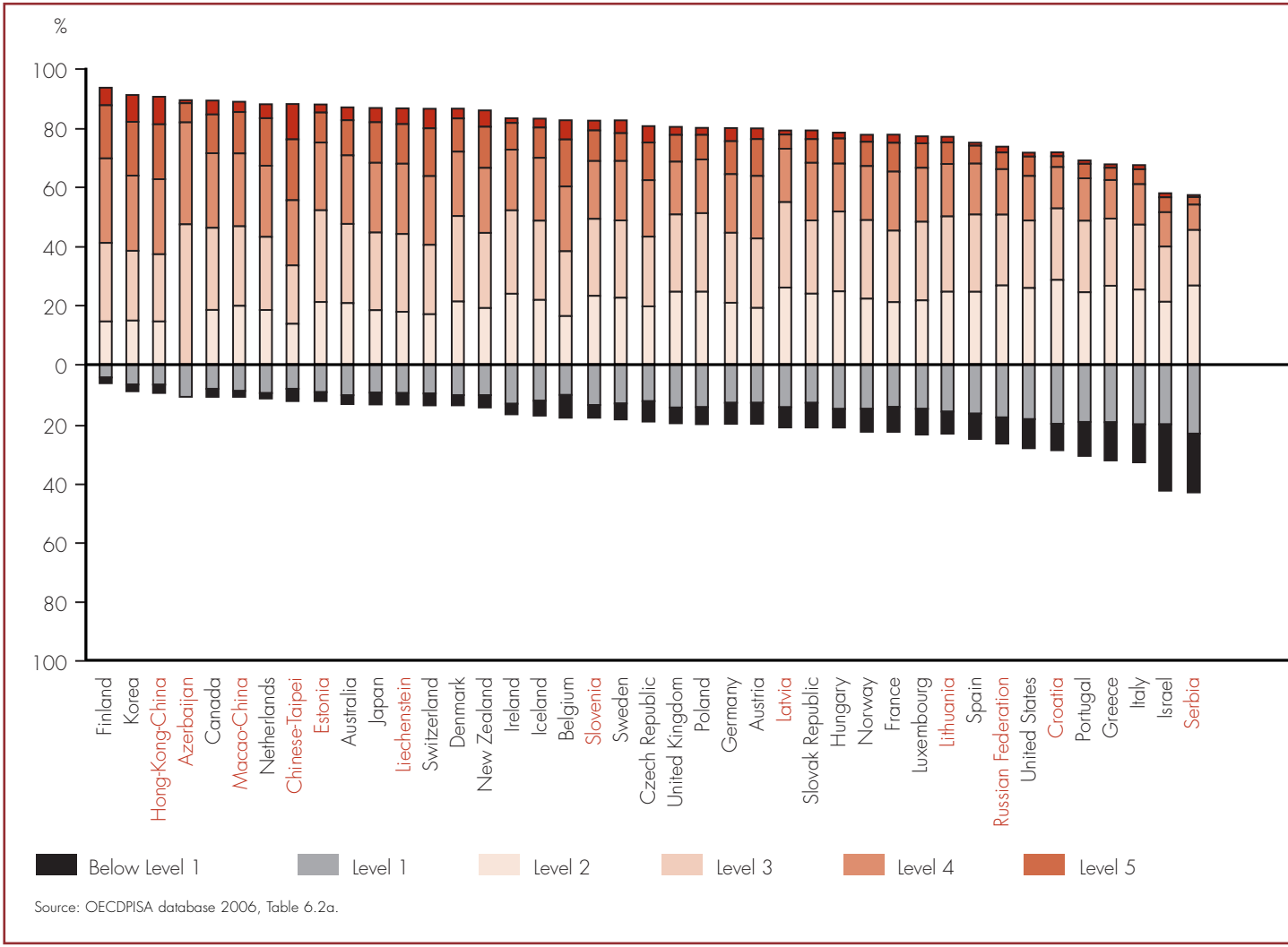
Argentina*, Australia, Austria, Azerbaijan*, Belgium, Brazil*, Bulgaria*, Canada, Chile*, Colombia*, Croatia*, Czech Republic, Denmark, Estonia*, Finland, France, Germany, Greece, Hong Kong (China)*, Hungary, Iceland, Indonesia*, Ireland, Israel*, Italy, Japan, Jordan*, Korea, Kyrgyz Republic*, Latvia*, Liechtenstein*, Lithuania*, Luxembourg, Macao (China)*, Mexico, The Netherlands, New Zealand, Norway, Poland, Portugal, Qatar*, Republic of Montenegro*, Republic of Serbia*, Romania*, Russian Federation*, Slovak Republic, Slovenia*, Spain, Sweden, Switzerland, Taipei (China)*, Thailand*, Tunisia*, Turkey, United Kingdom, United States, Uruguay*.

In the 2009 PISA, the following LAC countries were added: Dominican Republic, Panama, Peru, and Trinidad and Tobago.

The PISA covers the domains of reading, and mathematical and scientific literacy in all cycles. In this document, only the results for mathematical achievement are shown. Besides, an additional domain of problem solving was introduced in the 2003 PISA cycle to continue the examination of cross-curricular competencies.

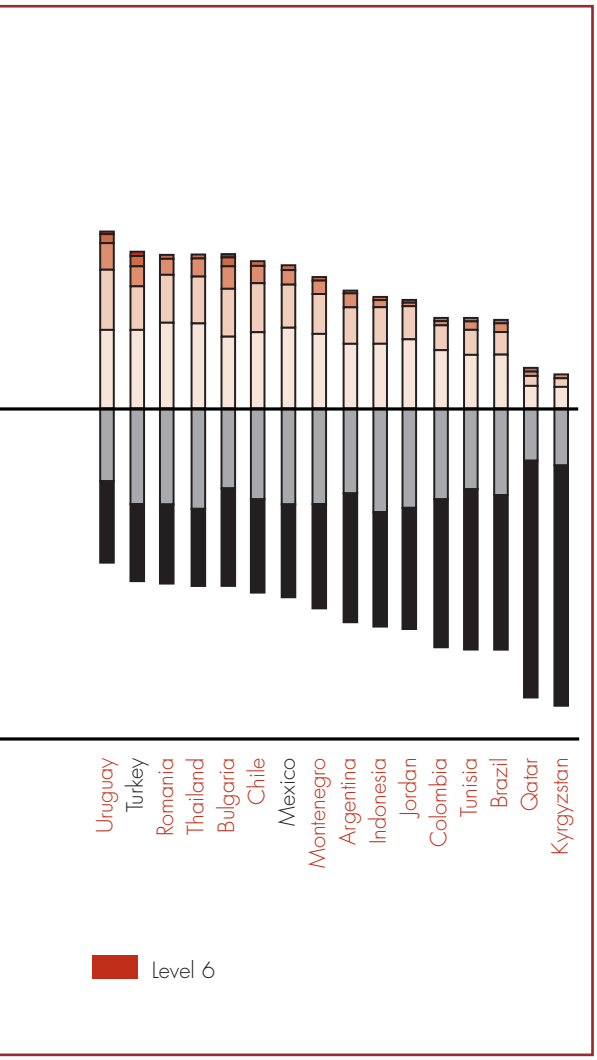
It is extremely important to analyze Figure 1, which shows the percentage of students at each proficiency level on the mathematics scale. Considering Levels 1 and below (which correspond to the bars below zero) among the six LAC countries that participated in this evaluation, Uruguay, the best of them, had nearly 50% of its students on those two levels. And now the question is, "What does that mean?" Let's look at the definition of Level 1.

- At Level 1, students can answer questions involving familiar contexts where all relevant information is present, and the questions are clearly defined. They are able to identify information and carry out routine procedures,



Source: OECDPISA database 2006, Table 6.2a.

Figure 1. Percentage of students at each proficiency level on the mathematics scale. Countries are ranked in descending order of percentage of 15-year-olds at levels 2, 3, 4, 5 and 6.



according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

Let's compare this with level 2:

- At Level 2, students can interpret and recognize situations in contexts that require no more than a direct inference. They can extract relevant information from a single source and make use of a single representational mode. They can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.

This means that most students in the six countries are below Level 2, so they do not have the ability to solve problems with basic algorithms, and are not capable of direct reasoning to solve a problem.

Various evaluations have shown that the situation in other LAC countries does not differ much from this.

What is happening in our classrooms?

The core of education is teaching, and this is done by teachers. Our mathematics teaching pool is inadequate to meet our current needs; many classes on this subject are taught by unqualified and under-qualified teachers. The only way to help our children understand and master mathematics is giving them teachers who are not only enthusiastic, but who also have a deep knowledge of their discipline and the professional training necessary to teach adequately. But teacher development is not a simple matter of training; it depends just as much —or even mostly— on continuing, high-quality professional development.

It is known that the ability to teach is not "something you're born with" (McKenzie report); it can be learned over time. The ability to discover which items are the hardest for students can only be acquired by training, mentoring, collaboration with peers, and practice. As we said before —but it is worth repeating it— teachers need to have a deep knowledge of their subject, since there is no substitute for this. We need to teach students not only what to learn, but also how to learn it.

A high-quality teacher encourages students to submit their work for criticism by their peers, and to reflect on how the arguments leading to conclusions were reached. High-quality teachers guide the class through knowledge, skills, and mathematics abilities.

It should also be noted that, in many countries, unsatisfactory mathematics results correlate with low language results. In Table 3 you will find the overall reading and mathematics results from the OECD's PISA 2006 database. The countries in bold are OECD members.

This suggests that efforts to improve mathematical literacy can be linked to efforts to improve language literacy. How can a student read and understand a

Table 3. Reading and mathematics results for selected countries (PISA, 2006)

Selected countries	Reading	Mathematics
Korea	556 (maximum)	547
Finland	547	548 (maximum)
Canada	527	527
Australia	513	520
Japan	498	523
United Kingdom	495	495 (average)
Germany	495	504
Slovenia	494 (average)	504
Hungary	482	491
Italy	469	462
Spain	461	480
Chile	442	411
Russian Federation	440	476
Uruguay	413	427
Mexico	410	406
Brazil	393	370
Colombia	385	370
Argentina	374	381
Kyrgyzstan	285 (minimum)	311 (minimum)

Source: OECD PISA database 2006.

problem if he lacks the necessary skills of his own language? The same is true for teachers. How can they communicate mathematical concepts without a good mastery of their language? The importance of this relationship is being more widely recognized by education experts.

Most LAC countries provide education for almost all their young people. Regrettably, the quality of such education is too often poor. Long-term solutions are urgently needed. The region needs better training for its mathematics teachers. In few words, the region needs: BETTER TEACHERS.

How can this be achieved? In many countries from the region, the responsibility for teacher training is scattered among different agencies, including Ministries of Education, teacher training colleges and programs, and to a lesser extent, university-level departments of mathematics. Within each country, including some English-speaking Caribbean countries, one may even find that all of these agencies train teachers for their national educational system. Therefore, on one hand, there may be too many agencies training teachers and working to improve their proficiency level, though they do not necessarily communicate much with each other.

On the other hand, except for a relatively small number of teachers trained by departments of mathematics, the role of mathematicians in the training process is very limited, and even non-existent in most of the region. This suggests that mathematicians can play a greater role in teacher education and training, provided that care is taken in their approach.

Indeed, if departments of mathematics throughout the region simply start trying to "improve" their teachers' training, without having a proper dialogue with them and summoning their cooperation, they risk fragmenting even further the process of teacher training. However, if departments of mathematics engage with the various agencies in their country, and establish a real dialogue with the teachers and summon their cooperation, there is a good chance that they can help unify and improve the process of training mathematics teachers for their country.

This second alternative, in which mathematicians establish meaningful partnerships with other agents, would allow the use of a mostly untapped resource to help improve mathematics teaching throughout the region. The aim of this document is to explore all possible actions that point in that direction.

3. MATHEMATICIANS AND MATHEMATICS EDUCATION IN LAC: SOME GOOD PRACTICES

In spite of our poor general results, we can find in the region examples of significant efforts that proved successful in improving mathematics teaching and results, and which involved professional mathematicians. The ICSU-LAC Scientific Planning Group on Mathematics Education compiled information from several countries. Obviously, the list is not exhaustive and surely enough, there are other good experiences besides the ones presented here. The order given here has no particular significance.

Argentina

Concrete efforts are being made in Argentina to promote mathematics, and capacity building on mathematics teaching. One experience that is worth looking at is the so-called International Problem-Solving Conference and Seminar (Seminario Internacional y Jornadas de Resolución de Problemas), which is held annually in coincidence with the National Mathematics Olympics (Olimpiada Nacional de Matemáticas). This annual event brings together experts from all over the world and more than 300 Argentinean teachers for a week. The Mathematics Olympics, which are one of the biggest and better mathematical competitions in the LAC region, deal with students that range from elementary school to high school.

The Universidad de Buenos Aires (UBA) brings mathematics to the general public through its Mathematics Basic Cycle (Ciclo Básico de Matemáticas). Each year, they organize Elementary Mathematics Exhibits (Muestras Elementales de Matemáticas) and receive school groups for a two-day visit. The students are toured around a mathematics exhibit and get the chance to play mathematical games prepared in advance. Lectures are also held for different target audiences, like teachers, students from different levels, and the general public.

Brazil

Although many efforts are made in Brazil, one of the country's most striking examples is the Brazilian Mathematics Olympics for Public Schools (Olimpiada Brasileira de Matemática das Escolas Públicas, OBMEP), which is led by professional mathematicians and has been held for the last three years: 2005, 2006, and 2007.

In this massive and amazing effort, around 17 million students from all over the country (even from the most isolated areas) took part in 2007. The OBMEP consists of three parts or sections, and as in all competitions, the first one includes exams, awards, and awarding ceremonies.

However, the distinctive features of these Olympics are certain initiatives that involve directly both the schools and the school mathematics teachers, in an effort to raise the general standards. The second and third parts of the OBMEP comprise: (i) Scholarship programs for 3 000 beneficiaries, a training program for teachers, and meetings of the gold medalists; and (ii) publications and supporting material.

The most important parts of the OBMEP are the scholarship program for outstanding students, and the 197 venues, in which they receive weekend training

for one year. Besides taking care of the students, the program also involves some teachers, for whom the Pure and Applied Mathematics Institute (Instituto Nacional de Matemáticas Pura e Aplicada, IMPA) offers a special program.

Colombia

The Colombian Mathematics Olympics (Olimpiadas Colombianas de Matemáticas) are probably one of the oldest mathematical Olympic programs in the region. This contest has run for nearly 30 years. In the last events, students from other parts of the region had been invited to train along with the Colombian students. This country promoted and organized the first Iberoamerican Mathematical Olympics. Besides, Colombia has participated in the Bolivarian Mathematics Olympics (Olimpiadas Bolivarianas de Matemáticas) since their beginning, back in 2000.

Cuba

Cuba is considered an exception in the LAC region. Teachers are trained at university level. Basic training for teachers is satisfactory in mathematics, and offers specific practices for mathematics teaching. The university-level Cuban Advanced Pedagogical Institutes Network (Red de Institutos Superiores Pedagógicos Cubanos) for teachers is very important for the island's educational system, specifically in the mathematical field. The island has solid mathematical curricula for mathematics and computer science teachers. Here, public policies for science and technology not only offer strong support for mathematics, but for science in general. According to Cuba's strategic plan, exact sciences are essential not only for technological progress, but also to strengthen education at all levels, and for capacity building among the new generations of Cuban scientists.

For Cubans, the need to involve the whole scientific community in order to strengthen education is clear, especially in regard to mathematics education, which is achieved by all kinds of means, even non-formal ones.

Chile

Chile has taken important and decisive steps to improve the quality of its mathematics teaching. One of the most relevant steps is that researchers and other prominent mathematicians, who previously were engaged exclusively on research activities and university-level teaching, are now strongly committed to increase the knowledge and training of school teachers.
When the TIMSS and PISA results were available, the diagnosis was:

- Great heterogeneity in mathematics teaching programs.
- Lack of connection between discipline knowledge and teaching techniques.
- Weakness in the training of teachers by disciplines.

Currently, Chile's mathematical community changed its general attitude —which previously tended to blame the educational system in general, and school teachers in particular for the country's low educational levels— in order to assume their share of the responsibility for children's failures, and to face this as a problem that needs to be solved by all the stakeholders. Some of the country's most talented mathematicians are taking steps to contribute to the solution.

Chile launched already a program of standards and certification for mathematics teachers at high-school level. This program offers special online courses for skill-building on mathematics teaching. A similar program for elementary school teachers is being prepared.

Costa Rica

This country has one of LAC's best elementary school systems. First of all, they focus on the teachers, whose training is done at universities. After regular high school, all future junior-high-school-level teachers go to college for three years, and then receive a two-year teaching training program. For instance, after three years at the university, any aspiring mathematics instructor can choose between two paths: becoming a mathematics teacher, or pursuing a B.A. in Mathematics. That way, knowledge on mathematics for future teachers seems to be guaranteed, and teacher training is pursued as a professional specialization.

Mexico

In Mexico, the situation is no better than in other LAC countries. Government evaluations show that the results are very poor. Clearly, the keystone for improving mathematics education is teachers' training. The Mexican Mathematical Society (Sociedad Matemática Mexicana) and the Mexican Academy of Sciences (Academia Mexicana de Ciencias) have started working on improving the teachers' mathematics knowledge.

For the first time in history, the Academy's scientists are approaching elementary and secondary-level school teachers through a program called Science in your School (La Ciencia en tu Escuela), in an attempt to change the prevailing attitude towards mathematics and sciences. This turned out to be a successful and good-quality program. However, it faces a major problem: how to reach out to nearly one million teachers. To upscale the program, the only solution seems to be using the internet; therefore, the program is starting to switch, from the classic classroom education, to a semi-virtual school system which is followed very closely by

counselors and scientists. The program La Ciencia en tu Escuela can be found at the Mexican Academy of Sciences web page (http://www.amc.unam.mx).

The Mexican Mathematical Society launched a program to improve the performance of 6 000 elementary school teachers whose students had poor evaluations in mathematics. Unfortunately, the program's academic level will not be the best possible, since the responsibility for teachers' training is shared by the National Education Workers Union (Sindicato Nacional de Trabajadores de la Educación, SNTE), the Ministry of Education, and the Mexican Mathematical Society.

Mathematical contests are a well-established tradition in Mexico. For example, the last Spring Contest (Concurso de Primavera) organized by the Mexican Academy of Sciences had almost 400 000 participants. Likewise, the Mexican Mathematical Olympics (Olimpiada Mexicana de Matemáticas), which is organized by the Mexican Mathematical Society, is quite successful at the international level.

4. SOME PROPOSALS FOR LATIN AMERICA AND THE CARIBBEAN: GENERAL OBJECTIVES

Most countries in the region have implemented policies to ensure that the coverage of compulsory schooling, up to the age of 16, gets as close as possible to 100%.

Despite some other efforts to improve their educational systems, LAC countries have failed to achieve significant results in mathematics. Hence, the main goal should be to improve the students' mathematical achievement in the region. The results of the international assessments performed in the few LAC countries that participate in such evaluations are discouraging, and need to be improved substantially.

In order to achieve this, the following detailed objectives are recommended.

- Reinforcing the link between professional mathematicians and school teachers. Few mathematics professionals are fully aware of current school practices or engage with the mathematical mainstream at other levels. Conversely, few teachers receive mathematical education at a sufficiently high level, and hence, most of them lack a complete vision of the field. Though active mathematicians are a potentially significant resource, on the whole they are not being brought into play as much as it would be desirable. By linking mathematics professionals with school teachers, improvements may be expected in the students' mathematical performance.

- Developing suitable data for performance evaluation. Currently, international evaluations such as those administered by the PISA or TIMSS give excellent assessments of mathematics performance for participating countries. However, relatively few LAC countries participate in such evaluations. Furthermore, these evaluations are limited to certain age groups, and hence, do not produce a detailed picture of students' performance as they proceed through their respective systems. By developing more complete data, a better understanding of the issues will be achieved, and potential solutions will be proposed.
- Defining sets of minimum standards for mathematics teachers at the elementary and intermediate school levels throughout the region. Presently, the qualifications required to teach mathematics at each level vary significantly throughout the region. Reasonable standards for mathematics teaching at each level should be pursued, based on current research and best practices.
- Using information and communications technologies (ICT) to give our teachers and students access to more resources. Although internet access rates in LAC are generally low (about 27%, as reported in 2007 by wdi , Word Bank), some countries have higher access rates, and this is growing fast in other nations. In addition, some countries with low general internet access rates have, on the contrary, a high percentage of schools with such access. Therefore, it can be assumed that, in the long run, a high percentage of students will have internet access. This should ensure that students and teachers have greater access to educational programs, supporting materials, and information to enhance their learning experience. The use of the internet will also allow the offering of a greater diversity of programs, and make them easier to tailor to the needs of each country.
- Finding strategies to improve simultaneously mathematics and language literacy. For most countries in the LAC region, language and mathematical literacy

rates are strikingly similar. This suggests that any improvements on one rate will be tied to progress in the other. Therefore, mathematics and language educators should be encouraged to work together, in order to develop strategies and programs that will improve simultaneously our mathematics and language literacy (Table 3).

- Continuing the involvement of mathematics education faculties in order to further support mathematics teachers in the fields of pedagogy and didactics. In order to improve the teachers' background, it is important that professional mathematicians not only transmit their mathematical thinking and knowledge to school teachers, but also help them improve their pedagogical and didactical skills by means of sensorial materials, teaching techniques, teaching resources for students with disabilities, etcetera.

5. SPECIFIC ACTIONS FOR A MATHEMATICS EDUCATION PROGRAM

How can the educational system get the right people to become teachers? How can it help them develop into effective instructors and raise the standards of every student?

To achieve this, it will be necessary to recruit the best students from different areas and get them interested in teaching. This can be done by raising the salaries to make them more competitive, but that is not the only determinant factor. In most developing countries, it has been shown that choosing teaching as a main career depends not only on the salary or culture, but also on a small set of simple but critical public policies: developing strict processes for teacher selection and training, compensating teachers with reasonable initial salaries, and giving a higher status to the teaching profession, among others. In general, countries that perform well on these factors are able to recruit teachers fresh from college, and then train them for one or two years. For instance, England made some strategic changes to modify the social status of the teaching profession; thus, in a five-year period, teaching became the most popular profession among undergraduate and graduate students.

One of LAC's biggest problems is that not even teachers master their subject, or even worse, they do not have a broad understanding of the subject they teach. One way to help them is putting people who master the subject in touch with

them, and trying to link professional mathematicians with mathematics educators and teachers.

School teachers, mathematics educators and mathematicians all need to know where to go, and how to proceed. It is necessary to define and implement teaching standards at all levels, and to develop supporting materials for both mathematics and pedagogy, especially for the initial training of teachers.

Similar actions must be implemented for in-service teachers, such as offering them supporting materials and summer programs, among others.

Linking mathematicians with mathematics education

A few good practices were noted before, in which mathematicians are actively engaged in efforts to improve mathematics education. An important action will be completing the assessment of the existing good practices, and sharing and expanding those experiences to other countries in the region.

In order to do this, the Regional Office should organize a workshop in which participating countries learn from good examples of interaction between mathematicians and educators. The aim of the workshop would be to explore the possibility of implementing similar actions in a larger group of LAC countries.

The shared good practices should also include examples of the use of ICT for linking mathematicians and educators, as well as improving the students' mathematical performance.

The creation of a mentoring program in which research mathematicians act as guides for new mathematics teachers, would be of great impact.

Developing suitable data

Many LAC countries perform national evaluations at several age groups. But to compare data among LAC countries, as well as internationally, all countries should be encouraged to participate in standardized international evaluations like PISA or TIMSS.

Another important issue is that these international evaluations are usually for children in the 8 to 15-years old age group. Since mathematics teaching requires diagnosing and correcting deficiencies at an early stage, it would be suitable to have reliable and comparable data to measure student achievement at age 12. Then, countries with reliable evaluations for 12-year-old children would be able to do long-term reforms to improve their performance.

Evaluations such as the ones proposed have significant costs, which may be a challenge for many national education budgets. This SPG proposes that ICSU creates a surveying team to undertake a feasibility study and to define feasible funding strategies.

Initial teacher training

Initial teacher training (ITT) refers to the conceptual and subject-specific set of knowledge on mathematics, and pedagogic skills, that should be expected from any new elementary and secondary school teacher, in order to increase his/her effectiveness in the classroom (i.e., having a broad understanding of mathematics and mastering their subject).

Once more, the proposals of this SPG will be focused on the link between mathematicians and educators. An especially important contribution would be setting regional standards for ITT, which is important in the same way as having standardized data for students' achievement that can be compared between

countries. It is also crucial to have minimum regional standards for ITT on mathematics.

As mentioned before, mathematics teaching, even at the elementary level, is a sophisticated process that requires deep knowledge of the subject. Professional mathematicians have the scientific knowledge to contribute to the establishment of minimum standards that will create a solid basis for training new teachers.

This SPG recommends that the Regional Office promotes and organizes regional forums, with the participation of mathematicians, educators and decision-makers, to initiate the discussion on regional standards for ITT on mathematics, and to explore ways for designing and implementing them.

In-service teachers

The transition from the initial training notion to that of continuing education is the landmark of modern pedagogy. There are many educational programs for in-service teachers, but regrettably, most of them are ineffective and oriented only to the didactic part, while most teachers need help in the area of mathematics. Such support must be of top quality, and has to be given by mathematicians who are not only interested in education, but who are also aware of the needs of in-service teachers.

Usually, this type of support works better when the teachers are convinced of the need to participate in such programs; therefore, it would be a good idea to offer the programs on an optional basis. If the programs are good enough, they will acquire a reputation and many teachers will try to enroll in them. Unfortunately, this leads to a big problem, because it seems that only while the programs are kept on a small scale, things work smoothly and standards are easily achieved.

One way to upscale this type of support is the use of computers. However, few LAC countries are ready for this kind of education, given that not all teachers know

how to use a computer or have access to one. Therefore, we need to start working on this. There are several possibilities for this type of education programs, such as: summer programs, weekend courses, and short-length curricula. Given the experiences had in some LAC countries, one successful way to organize the programs is using the weekends during the academic year, and then having a one-week-long summer camp. In addition, it is very important to give teachers support, once or twice a week, during the academic year. This can be done for students who are finishing their B.A. in mathematics. To provide more support to the use of computers, it would be useful to create a website where teachers could ask questions and discuss their experiences with other peers. Mathematicians would also need to prepare supporting material tailored to the teacher's needs.

All of these features are present in programs with relatively small audiences. Hence, the question of upscaling them remains in the air. As stated before, one way to do it would be the use of computers, but this alternative is not as efficient as regular programs. Nevertheless, offering these programs on the internet would be an important step. This will require a large number of instructors to help teachers via computers. And one way to prepare those instructors would be selecting teachers who have been successful when training students for Olympics, and start working with them.

School-based national Olympics

Many LAC countries have had good experiences with Mathematics Olympics. However, many of their models are focused exclusively on the selection of students to compete in international Olympics. The new proposal puts emphasis on increasing the quality of mathematics teaching in all schools, and the use of competitions to connect professional mathematicians with students and teachers at their schools.

The main objectives of this proposal are:

- Affirming and reaffirming excellence as a major value for public education
- Increasing the students' interest in mathematics in order to improve its teaching in public schools.
- Improving the results in mathematics achievement.

Therefore, this proposal should encourage each country to establish school-based national Olympics focused on reaching every school. For that, it will be absolutely necessary to work out a partnership between national governments and the leading academic institutions of each country, since this would ensure that highly qualified mathematicians are committed to the project. As other Olympics, school-based national Olympics are competitions that feature exams in several stages, with awards going to the top performers. However, these school-based national Olympics would have these additional characteristics:

- Students should register through the school, not individually.
- Extended training programs should be given to all medalists, at all levels, endowing them ideally with scholarships. These programs should be made available nationwide.
- Extended training should be given to all identified successful mathematics school teachers. These training programs should be supported academically by professional mathematicians, and should be cost-free for the teachers.
- Professional mathematicians should produce support materials to help students and teachers prepare for the exams.
- Professional mathematicians should set the examination questions and assume the responsibility for their marking, whether personally or helped by trained assistants.

Many challenges will be faced while implementing this fully, and it may take up to three international evaluations to demonstrate the effectiveness of the program. In this regard, it is recommended that countries willing to start school-based Olympics consider supporting those programs for at least 10 years.

Backed by ICSU's endorsement of the proposal, the SPG recommends that a surveying team be convened to identify a few countries willing to participate in a pilot program, and to help them implement it, especially in regard to funding. Meetings with all stakeholders should also be considered.

Financially supported sabbatical semesters for mathematics teachers

The possibility for teachers to discontinue their professional practice during one semester in order to update themselves will be invaluable. During that period, they could study at any Mathematics Institute in LAC, or at the headquarters of any National Olympics in the region, where they would be exposed to significant experiences in mathematical reasoning, as well to a series of basic programs from an advanced point of view. To be eligible for such program, teachers would be required to submit a project involving deep mathematics, depending on what they want to do and where, and a selection committee should award financial support to cover the sabbatical period. In the report called Foundations for Success (2008), the panel stated that the need for teachers to develop stronger mathematics skills is self-evident, but also that more research must be done in order to find the best ways to prepare those teachers. One possibility could be this sabbatical program.

6. CONCLUSIONS

As mentioned in the introduction: Mathematics education can no longer be seen only as a human right for a minority of students who will pursue scientific-based careers in the future, or for especially gifted or motivated students, or exclusively for males. Mathematics education is now understood as a basic right for all students, and as a specific type of training for life. More than half of the students in the LAC region are below PISA Level 1, which means that most of them cannot identify mathematical information, nor carry out routine procedures, according to direct instructions in explicit situations.

We need to act urgently. Although some countries have had successful experiences when trying to reverse the depicted situation, this is not enough. Besides, such experiences are not common to all countries. It would be better if we could get all LAC countries to act together, since their problems are similar. This report identifies some actions that can be multiplied or introduced across the region.

In all of them, professional mathematicians across the region should help teachers acquire the necessary mathematical knowledge and become more effective instructors, able to raise the standards of every student.

This is URGENT; and something must be done.

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GLOSSARY OF ACRONYMS

ABC: Brazilian Academy of Sciences
CMM: Center for Mathematical Modeling, Chile
CONACYT: National Council on Science and Technology, Mexico
GDP: Gross Domestic Product
ICSU: International Council for Science
ICSU-LAC: ICSU Regional Office for Latin America and the Caribbean
ICTs: Information and Communications Technologies
IMPA: National Institute of Pure and Applied Mathematics, Brazil
ITT: Initial Teacher Training
LAC: Latin America and the Caribbean
OECD: Organization for Economic Co-operation and Development
PISA: Program for International Student Assessment
RCLAC: Regional Committee for Latin America and the Caribbean
SPG: Scientific Planning Group
TIMSS: Trends in the International Mathematics and Science Study
UBA: University of Buenos Aires, Argentina
USD: United States Dollar


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