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MATHEMATICS FOR ACTION

Supporting Science-Based Decision-Making

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SHORT SUMMARY

Mathematics empowers sustainable development

Everything we do is based on some mathematical structure, and although mathematics is often considered abstract, it is fundamental to how we understand nature, the larger universe, with its time and space dimensions and a myriad of uncertainties.

The Covid-19 pandemic brought mathematical modeling to the forefront of public attention and debate. Vocabulary such as 'flattening the curve' has become part of the collective lexicon. Governments all over the world rely on mathematics not only to forecast the epidemic but also to understand social issues like vaccine hesitancy.

Mathematics has allowed for pivotal improvements in weather prediction and has applications in agriculture and fisheries. **With new mathematical approaches, a tropical cyclone's track can now be predicted up to 1 week in advance giving communities time to evacuate, and potentially saving lives and reducing economic losses.**

41%

of the global population is at risk from tropical cyclone flooding.

The *Mathematics for Action* toolkit focuses on engaging stories of mathematics *in* action. Written by mathematicians and thought leaders from across the globe, it presents fascinating research of how mathematics is addressing the world's most pressing challenges.

The toolkit provides insightful information for decision-makers and for all those who seek proofs to challenging questions and it presents new avenues for scientific research.

"Since wars begin in the minds of men and women it is in the minds of men and women that the defences of peace must be constructed"

MATHEMATICS FOR ACTION

Supporting Science-Based Decision-Making

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(eds)

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Centre International de Mathématiques Pures et Appliquées (CIMPA)

European Mathematical Society (EMS)

Institut National des Sciences Mathématiques et de Leurs Interactions (CNRS)

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International Commission on Mathematical Instruction (ICMI)

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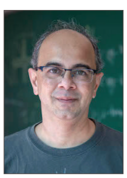

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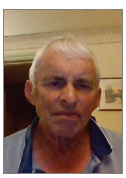

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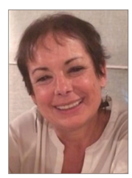

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FOREWORD

This toolkit showcasing *Mathematics for Action* comes at a time when mathematics is becoming an increasingly precious tool for decision-makers. A growing range of mathematical models are enabling us to analyse the extent to which natural phenomena and those we have engendered ourselves will affect how we live and whether we manage to sustain our increasingly fragile environment. This toolkit is UNESCO's way of drawing global attention to the need for public policies to be based on evidence which, increasingly, will stem from basic research.

But why should UNESCO be producing such a publication? Simply because UNESCO is the only United Nations agency with a mandate for mathematics. This mandate for mathematics is as old as the S for science in UNESCO's name, which dates from the Organization's founding in 1946.

In 1962, UNESCO founded the Latin American Centre for Mathematics (CLAM) in Buenos Aires, Argentina, in recognition of the need for a solid grounding in mathematics in the developing world. This was an exciting time for mathematics. A few years earlier, the first artificial satellite had been sent into in space. A few years later, human beings would walk on the Moon for the very first time. The term artificial intelligence was coined in 1956, three years before the first microchip was patented. Over the coming decades, the miniaturization of integrated circuits would make it possible to manufacture ever-smaller mechanical, electronic and optical devices. Today's smartphones use millions of miniscule transistors to perform complex processes. These smartphones wouldn't have been possible without mathematicians.

UNESCO was also behind the establishment of another institution which has trained mathematicians and fostered research in national institutions around the world over the past few decades. I am referring to the International Centre for Pure and Applied Mathematics in Nice, France, established in 1978.

UNESCO has devoted a great deal of its work to improving the quality of mathematics education and research but remains something of an enigma to the person in the street. Everyone recognizes, for example, that mathematics is omnipresent in today's world – notably in the technological items all around us and in exchange and communication processes – but this presence is generally not in evidence. This makes it difficult for some to see the point of developing a mathematics culture beyond basic skills in numeracy, measurements and calculation.

This is why it is important for basic education to bring mathematics to the fore. This is especially vital because 'mathematical literacy' requirements far exceed needs traditionally associated with basic computational knowledge. Mathematics is still often perceived as an almost exclusively solitary activity, cut off from the problems of the real world and independent of technology. Furthermore, mathematics is often still seen as a purely deductive activity in which perfectly rigorous formal proofs are used to produce theorem after theorem. These many misunderstandings affect the teaching of mathematics by raising barriers to quality mathematics education for all.

This is why UNESCO supported the World Mathematical Year in 2000, in order to familiarize people around the world with the impact of mathematics on their daily lives. This is why a team led by UNESCO designed a travelling exhibition for the general public in 2004 called Experiencing Mathematics.

This is why the present toolkit has been designed for policy-makers. *Mathematics for Action* demonstrates how mathematics lies at the heart of the evidence-based policies that governments around the world adopt on a regular basis to tackle a particular socio-economic or environmental issue.

Shamila Nair-Bedouelle
Assistant Director-General for Natural Sciences

INTRODUCTION

Today's world faces a daunting array of complex and interconnected challenges. Issues such as food insecurity, inequality, infectious agents, climate change, land degradation, biodiversity loss, mass migration, conflict, and political unrest pose obstacles to development and put societies at risk worldwide. Moreover, projected population growth patterns and climate change impacts will intensify these challenges.

MATHEMATICS OF PLANET EARTH

Nearly a decade ago, the mathematics community launched the Mathematics of Planet Earth 2013 (MPE2013), a year-long initiative to showcase the ways in which mathematical sciences can be useful in tackling these global problems. Over the year, MPE2013 grew into an international partnership of more than 150 scientific societies, universities, research institutes, and professional organizations. MPE2013 underscored the multidisciplinary nature of the problems facing the planet and emphasized multidisciplinary partnerships to address these problems.

At the end of 2013, a new structure was designed to support ongoing research efforts and maintain the initiative's momentum. This publication — *Mathematics for Action: Supporting Science-Based Decision-Making* — is one of the many outcomes of these efforts.

Mathematics for Action is a collection of briefs highlighting the role of mathematics in addressing issues of global relevance. Written by 32 mathematicians and thought leaders from across the world, the 26 briefs showcase three types of topics:

- **Success Stories** — Mathematical concepts and tools that advance solutions to everyday problems, such as monitoring and predicting the spread of epidemics;
- **Mathematics Illuminated** — Mathematical concepts that help us understand and describe real-world processes;
- **Grand Challenges and Opportunities for Mathematics** — Pressing problems that mathematics can help solve, from food system resilience to climate change.

While many of the briefs share common themes or concepts, each brief can be read independently or out of sequence. Together the collection emphasizes the strength and potential of the mathematical sciences to meet global challenges, and highlights opportunities and innovative approaches that may have broader applicability to science-based decision-making.

TOOLKIT ROADMAP

The 2030 Agenda for Sustainable Development, adopted by all United Nations Member States in 2015, provides a shared plan of action for people, the planet, prosperity, and peace. The 17 Sustainable Development Goals stimulate action over the next 15 years in areas of critical importance for humanity. The 26 briefs are organized by these goals and address 11 of the 17 goals:

Goal 1: End poverty in all its forms everywhere. *Visualizing Poverty* details mathematical techniques for collecting and mapping poverty data — techniques as accurate and more efficient than traditional survey-based methods.

Goal 2: End hunger, achieve food security and improved nutrition and promote sustainable agriculture. *Strengthening Food Security* describes mathematical approaches that can help identify shocks and design optimal mitigation and adaptation strategies for building food-system resilience.

Goal 3. Ensure healthy lives and promote well-being for all at all ages. Five briefs address topics related to the Sars-CoV-2 pandemic. ***Modeling Infectious Diseases*** provides the foundation for how infectious diseases are mathematically modeled and what can be learned from these models. ***Harnessing the Power of Data*** details some of the mathematically-grounded and locally-nuanced pandemic response efforts underway in Africa. ***Improving Pandemic Forecasts*** describes how a state-of-the-art technique from weather forecasting was used to enhance the prediction accuracy of

COVID-19 models. ***Enhancing Vaccine Design*** describes the novel ways mathematics has helped accelerate the design, testing, and monitoring of new vaccines, including the Sars-CoV-2 vaccines. Finally, ***Modeling Vaccine Hesitancy*** considers the phenomenon known as the free-rider problem in the context of vaccine hesitancy, and what this means for decision-makers.

Goal 4: Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all. *Teaching Mathematics* reflects on the importance of mathematics education and the role of mathematics teachers in improving students' learning outcomes and socioeconomic mobility.

Goal 5. Achieve gender equality and empower all women and girls. *Tracking Gender Parity* examines the mathematical and statistical underpinnings of indicators used to measure and track the legal, economic, social, and cultural factors contributing to a gender gap.

Goal 6. Ensure availability and sustainable management of water and sanitation for all. *Managing Water Resources* highlights how a statistical tool known as Bayes Theorem can be used to quantify risks and identify appropriate options for the management of water supplies. *Shifting Lake Turbidity* highlights how mathematics can provide insights into the mechanisms that drive shallow lakes from clear to turbid and support effective, cost-efficient, and sustained approaches to restore clarity.

Goal 9. Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation. *Reckoning with Uncertainty* shares lessons for responsible mathematical modeling that can help society demand the quality it needs from mathematical modeling.

Goal 11. Make cities and human settlements inclusive, safe, resilient and sustainable. *Preparing for a Crisis* deals with the resilience of digitized systems and describes some of the mathematical methods and tools that have proven invaluable for addressing vulnerabilities in critical systems and processes and building more resilient systems and societies.

Goal 12. Ensure sustainable consumption and production patterns. *Valuing Natural Capital* considers efforts to integrate the value of ecosystem services in national development frameworks and describes the role of mathematics in strengthening these efforts. ***Allocating Scare Resources*** illustrates how mathematics can support integrated approaches to food-energy-water nexus management and decision-making, including model-based solutions to prioritize and optimize investments.

Goal 13. Take urgent action to combat climate change and its impacts. Five briefs address climate change, many aspects of which are intertwined in mathematics. ***Modeling Climate*** highlights the energy-balance model, a simple but powerful mathematical model that can help governments, policymakers, and the public understand Earth's past, present, and future climates. ***Facing Future Climates*** describes how mathematical models can provide information for policy-relevant and regionally-specific decision-making, allowing countries to scale up and accelerate adaptation and disaster reduction activities. ***Forecasting Cyclones*** describes how mathematical models are used to predict the path and intensity of tropical cyclones and their projected impacts. ***Attributing Extreme Weather*** details the new science of event attribution, which has enabled scientists to make quantitative statements about the influence of human-induced global warming on specific individual extreme weather events. Finally, ***Pinpointing the Indian Monsoon*** describes how mathematical models are used to predict the arrival, intensity, and duration of the Indian Summer Monsoon, a phenomenon vital to Indian society, agriculture, tourism, and economic development.

Goal 14. Conserve and sustainably use the oceans, seas and marine resources for sustainable development. *Sustaining Fisheries* describes newer integrated mathematical models that capture the economic, social, and ecological drivers of fisheries and promise improved support for sustainable fisheries management and decision-making.

Goal 15. Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss. *Measuring Biodiversity* looks at quantitative diversity indices and describes innovative mathematical tools for choosing these measures and gathering and processing biodiversity data. ***Listening in on Wildlife*** describes innovative mathematical techniques that can provide a rapid, efficient way to process wildlife sound data and ultimately better support biodiversity conservation efforts. ***Battling Invaders*** highlights mathematical models that can help scientists predict the impact of invasive species on native species and quantify efforts required to control and eradicate damaging invasive populations.

Goal 16. Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels. *Preserving Privacy* deals with federated learning, a new mathematical technique that supports building models trained in a distributed fashion such that private data never leaves a given participant or institution. This advancement will have significant ramifications for medicine, banking, and other areas where data privacy is paramount. Finally, ***Finding the Missing*** describes how complex networks can help support searches for people who go missing in connection with armed conflicts, other situations of violence, migration, and natural disasters.

While the 26 briefs cover a variety of mathematical applications, the collection is by no means exhaustive. But it does give an indication of the many diverse ways that mathematics can empower sustainable development.

> **Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them.**
>
> — Joseph Fourier,
> French mathematician and physicist

1 NO POVERTY

VISUALIZING POVERTY

AI-POWERED MAPS IMPROVE ESTIMATES AND PREDICTIONS

Eradicating poverty in all its forms remains one of the world's most urgent challenges. While much of the globe has experienced a decrease in extreme poverty, there were still about 700 million people living on less than $1.90 a day in 2021. For many countries, the ability to effectively assist their poor citizens is challenged by poverty data that lacks detail, is outdated, or inaccessible. Using mathematical approaches, analysts can enhance existing poverty estimates by integrating cell phone, satellite, Facebook connectivity, and other nontraditional data sources with conventional data. These enhanced poverty estimates will help decision-makers design more effective and efficient strategies to eradicate poverty by 2030.

KEY MESSAGES

- ☑ Poverty maps provide detailed estimates of poverty presented in the form of maps. Maps are not only highly effective visual communication tools, but they preserve spatial relationships among different areas, which would be impossible in a tabular data format.
- ☑ Reliable poverty data is often not available at fine geographical levels due to limitations in conventional surveying methods. To help fill gaps, nontraditional data sources can serve as proxies for where impoverished people live. Using mathematical approaches, this data can be combined with traditional data sources.
- ☑ The aggregated estimates produce spatially-fine poverty maps that enable decision-makers to channel aid more precisely to the poor.
- ☑ By increasing the granularity of poverty estimates, decision-makers can assess regional variations in poverty and growth. This allows for more effective targeting and prioritizing of policy interventions and resources based on local conditions.

Globally, about 700 million people — or 11% of the world's population — live in extreme poverty. While progress has been made in many regions, poverty in all its forms remains a persistent and prolific problem, particularly in sub-Saharan Africa. The Covid-19 pandemic, too, challenged some gains by pushing an additional 97 million people into poverty – a historically unprecedented increase in global poverty.

To achieve the Sustainable Development Goal of eradicating poverty in all its forms by 2030, decision-makers need more and better quality data to effectively target the challenges and barriers sustaining poverty. Too often, the data needed to design effective and efficient poverty-reduction policies and programs are inadequate at best and non-existent at worst.

TRACKING POVERTY

Traditionally, policymakers have relied on household surveys of income and expenditure or living standards to track and measure poverty. However, while such surveys have sample sizes large enough to provide nationally-representative samples, they're typically not large enough to provide reliable estimates at more granular levels such as villages or states.

Some countries complement household survey data with national census or administrative data to increase granularity. Analysts use multiple regression analysis to build a mathematical model of welfare or household consumption using variables available in both datasets. The model parameters are then applied to the census data to provide an estimate of consumption per capita for each household. These estimates are then aggregated to estimate poverty at a small-area level — village, state, or municipality level, for example. Finally, the estimates are typically merged with a map using a geographic information system (GIS). The maps concisely summarize large amounts of poverty data for hundreds or even thousands of small areas on a single page and in a readily-understandable, visual format. A map format also improves the interpretation of poverty data by preserving the spatial relationships between areas, which would not be possible with a conventional tabular format. Countries have used poverty mapping tools to deepen their understanding of poverty and its determinants, examine geographical, natural and climatic determinants of poverty, target resource and funding allocation, and analyze existing programs and policies and assess their effectiveness.

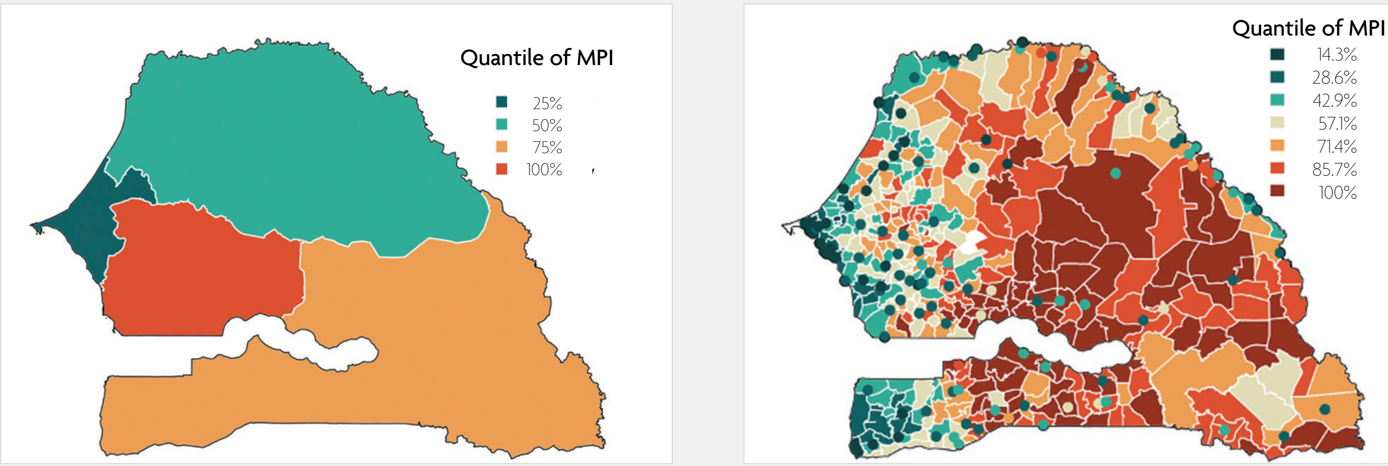
FILLING THE GAPS

Household survey data is costly and time-consuming to collect and assemble, and in developing countries, national census data is often out of date or inaccessible. Over the last few years, researchers have explored the use of nontraditional data sources to help fill information gaps in

CASE STUDY: HARNESSING BIG DATA TO IMPROVE SENEGALESE POVERTY MAPS

Researchers used a machine learning-based framework to combine cellphone records with satellite imagery and GIS data to create maps detailing poverty levels in 552 communes in Senegal. The cellphone dataset contained 11 billion calls and texts from more than 9 million Senegalese mobile phone users. Anonymized call information captured details on people and their movements. Data from satellite imagery, geographic information systems, and weather stations provided insights into poverty indicators such as the presence of electricity, paved roads, agriculture, and other signs of development. The enhanced estimates were then validated using the concurrent census data.

The researchers' "big data" map visualizes poverty variations not evident in the existing poverty map, which only divides the country into four regions. For example, the updated map revealed that communes in the country's interior have higher poverty levels. The capital city Dakar and surrounding communes in the west and along the coastal boundary have less poverty than the rest of the country. And, unlike conventional methods, their maps can be generated frequently and cost-efficiently.



On the left: The 2016 poverty map of the Global Multidimensional Poverty Index (MPI) for four divisions of Senegal (West, North, South, and Center). On the right: Updated poverty map of MPI for Senegal's 556 communes, created using big data. Urban centers are depicted as small circles. *"Combining disparate data sources for improved poverty prediction and mapping" by Neeti Pokhriyal and Damien Christophe Jacques is licensed under CC by 4.0.*

poverty data. By applying mathematical tools like machine-learning algorithms to cell phone records, satellite imagery, GIS data, and social media connectivity data, they can supplement traditional poverty data sources to build micro-estimates of wealth and poverty. These nontraditional sources can serve as proxies for identifying lagging development in particular areas. For example, nighttime lights provide an excellent proxy for economic activity by revealing the presence of electricity. By comparing daytime and nighttime satellite images, machine learning models can predict relative levels of prosperity. Similarly, cell phone data such as international call volume, contacts, and daily travel distance can provide an accurate and timely indication of living conditions. The mathematically aggregated estimates are then used to rank villages and communities by indicators of poverty and wealth and produce maps.

CONCLUSIONS

Big data-enhanced maps have led to increased demand for more independent and localized measurements of poverty. In many countries, decisions on the Sustainable Development Goals are being made using poverty data that is neither granular enough nor timely to design and deliver effective policies and programs that benefit the poorest of the poor. For example, household survey data hides the often-different experiences of women. These new mathematical approaches allow for disaggregation of poverty data by location, gender, age, and income as required for the Sustainable Development Goal indicators. The ability to disaggregate data will be equally valuable for fulfilling other goals and indicators.

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STRENGTHENING FOOD SECURITY

FOOD SYSTEM RESILIENCE AND SUSTAINABILITY

According to the World Food Program, in 2019, 135 million people suffered from acute hunger. This number continues to rise, largely due to global shocks ranging from extreme weather to conflicts and insecurity, desert locusts, and the COVID-19 pandemic. These and other shocks disrupt food supply chains and make it even more difficult for food-insecure people to meet their needs. Building sustainable, productive, and resilient food systems is critical to reversing trends and ending hunger by 2030. Mathematical approaches can help identify shocks that influence food systems, advance research-based strategies that build food system resilience, and ultimately improve food and nutrition security for all.

KEY MESSAGES

- ☑ Conflicts and insecurity, extreme weather, epidemics, and other shocks that influence food systems disrupt food supply chains and threaten food security. Mathematical techniques can help evaluate both risks and responses, and ultimately make food systems more resilient to current and future shocks.
- ☑ Mathematical models can identify optimal solutions for management objectives, such as minimizing costs, capturing market feedback, or simulating policies.
- ☑ Multiple risk scenarios can be examined through a mathematical lens. Risk assessments inform the development of mitigation and adaptation strategies.
- ☑ A food system is a complex network that is deeply associated with health, society, and the environment. Understanding this network — made up numerous actors and relationships — is a grand challenge for mathematics. Responding to this challenge can help improve food and nutrition security for all.

Food systems are the agricultural chains of market and non-market functions and actors connecting 'farm to fork.' They include producers, transporters, packagers, processors, consumers, and markets, all connected by the circulation of food. While they operate within the context of economics, food systems are subject to both biophysical and socio-economic constraints.

While food systems have produced more and better food over the past 100 years, they've become more vulnerable to shocks and disturbances. Today, stresses to food systems from population growth, increasing urbanization, climate change, agricultural pests, and disease threaten human well-being. Understanding food system stresses and interventions can help make society more resilient to current and future shocks.

FOOD SYSTEM SHOCKS

Food systems are exposed to many different disturbances — some known, many unknown, or even unknowable. Some, like an interruption of trade routes or political upheaval, happen on a relatively short time scale. Others occur over a more extended period — invasive species and livestock diseases, for example — or like droughts, extend over even longer periods of time. Disturbances may be local or regional, but their effects are often felt globally and have implications for social and economic development, Inequality and social justice, and human health.

During the 2008 global food crisis, prices of basic foods like wheat and rice rose dramatically. The causes for the crisis included droughts, price increases for fuel and fertilizers, changes in food demands, and speculative trading in the world markets. It led to riots in many developing countries.

Specialization and concentration along the supply chain increases a food system's vulnerability to shocks and disturbances. Boosting the diversity of a food supply chain buffers against these shocks. Mathematical techniques for quantifying risk-mitigation approaches have already been developed and applied in many areas, from engineering to finance.

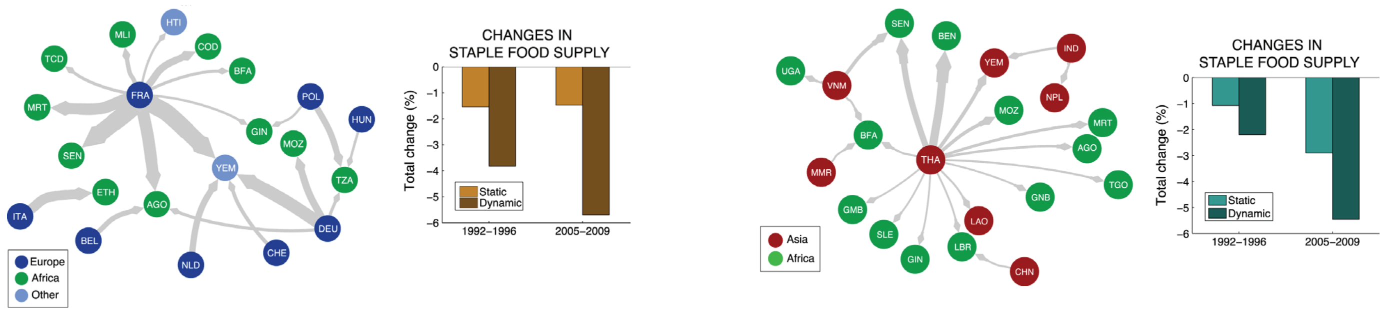
MATHEMATICAL MODELS

Food systems are demand-driven and involve personal choices and cultural preferences at many levels. For such a complex system, it's practically impossible to come up with simple numerical indicators, such as a failure probability or an expected cost. Instead, researchers use mathematical models that focus on particular features of the system. These models incorporate internal dynamics as well as external influences, and they may be applicable even for disturbances that have never been observed. They can account for uncertainties, such as weather, price fluctuations, or consumer behavior.

Mathematical models identify optimal solutions to a

NETWORK DIAGRAMS OF GLOBAL WHEAT AND RICE TRADES

IMPACTS OF WHEAT DISTURBANCES | **IMPACTS OF RICE DISTURBANCES**



Trade connections in 2009 and staple-food supply losses averaged over 1992–1996 and 2005–2009 for least developed countries. Each country is represented by a node and trade direction (i.e., exports) are depicted by an arrow. Line widths are proportional to the 2009 trade flow volumes. *"Assessing the evolving fragility of the global food system" by Michael Puma, Satyajit Bose, So Young Chon, and Benjamin I Cook is licensed under CC by 3.0.*

specified objective function, for example, minimizing the cost of meeting nutritional needs or simulating antibiotic resistance control policy in agriculture. Computable general equilibrium models, one class of mathematical models, have been used to capture market feedbacks for prices and quantities in response to system changes. Mathematical and statistical models are sometimes used to explore the potential consequences of policies or decisions and guide the timing and implementation of interventions to prevent or mitigate future adverse events.

MATHEMATICAL PERSPECTIVES

Several mathematical concepts can lend perspectives to food system behaviors and risks:

- A **tipping point** is a critical point beyond which a system undergoes a significant change that is difficult to reverse. With monoculture farming, for example, a single disease or pest outbreak can potentially collapse an entire food system.
- **Extreme events** occur near the upper or lower limit of typical observations and thus carry significant consequences for the system. For example, an extended period of extreme drought has created ideal conditions for grasshopper eggs to hatch. As a result, dense swarms of adult grasshoppers have now eaten large swaths of cropland and denuded trees across western North America.
- **Networks** are systems of sites, or nodes, with connections or links between them. Nodes and links may have extra properties which may change over time. Network models can be used to describe food system transportation and trade. Additional properties of network sites and connections, such as storage capacities or travel times, are easily incorporated and updated as they change over time. Many of these models were developed for application in other areas, such as air traffic and logistics.

EXPLORING SCENARIOS

In 2015, Lloyd's of London explored a scenario of a dramatic disruption to global food production and its consequences. In their scenario, El Niño conditions — from drought in Australia to severe flooding in Southeast Asia and crop reductions in South America — cause massive amounts of food crops to die in fields worldwide. Globally, corn, soybean, and rice production fall by upwards of 10 percent. Soaring food prices coupled with civil unrest would follow, leading to widespread economic, political, and social impacts, predictably hitting poorer countries the hardest. Mathematics can help identify likely pathways into and out of such a compound crisis.

Network science has also developed tools to describe the evolution of trade networks over time. By quantifying rates at which trade volumes change, and connections are established or disappear, researchers can anticipate pathways into and out of food crises.

CONCLUSIONS

Food systems are complex networks with unlimited actors and relationships. Modeling such a complex system remains a grand challenge for researchers. Ultimately, mathematics may hold the key to a better understanding of food systems and thus the well-being of future generations.

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MODELING INFECTIOUS DISEASES

FORECASTING THE SPREAD OF AN EPIDEMIC

A simple mathematical model can capture the hallmark curve of typical infectious disease epidemics. During the initial spread, the number of infections increases exponentially. The curve peaks and eventually flattens out when the pool of uninfected is sufficiently depleted. Such models allow scientists to estimate a measure of the contagiousness of a pathogen, making it possible to project when the number of simultaneous infections will peak and the severity of that peak. Forecasts based on mathematical models help inform public health decisions about pandemic planning, resource allocation, and implementation of social distancing measures and other interventions and policies to contain the spread of disease.

KEY MESSAGES

- ☑ Mathematical models provide invaluable tools for public health decision-making, both by forecasting the likely impact of an epidemic and by predicting the effectiveness of measures of disease containment and prevention.
- ☑ When a pathogen's basic reproduction number (R0) falls below 1 in a given place, infection spread slows and the epidemic can be controlled in that area. Physical distancing measures have proven to reduce the value of R0.
- ☑ When most of a population is immune to an infectious disease, those who are not immune are provided indirect protection, or herd immunity. Depending on how infectious the pathogen is, 70% to 90% of the population would need to be immune to establish herd immunity.

SEIRD MODELS

For diseases like COVID-19 and seasonal influenza that spread through person-to-person contact, the most popular type of epidemiological model subdivides the population into classes of individuals or compartments. In the SEIRD model, these are **s**usceptible, **e**xposed but not yet contagious, **i**nfected, **r**ecovered, and **d**ead. The model also contains a description of the mechanisms by which people move from one compartment to the next, which can then be used to predict the future spread of the corresponding disease.

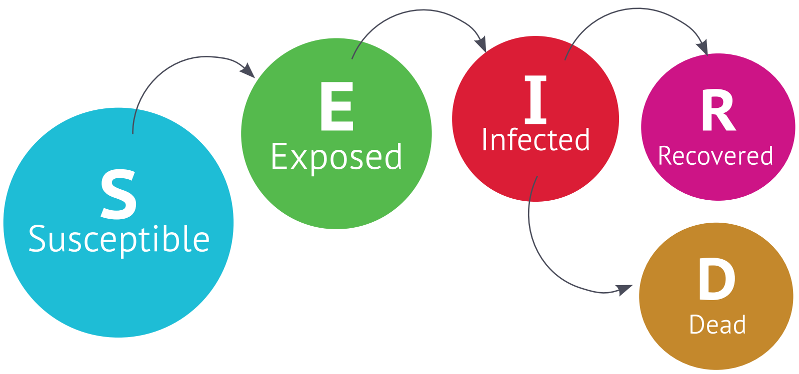
The SEIRD model relies on four key parameters:

- **Basic Reproduction Number, or R-Naught (R0)**. The number of people that a single infected person can be expected to transmit the disease to.
- **Latent or Incubation Period**. The time an infected individual is not yet contagious.
- **Infectious Period**. The length of time an individual is contagious.
- **Infection Fatality Rate.** The percentage of infected who do not survive.

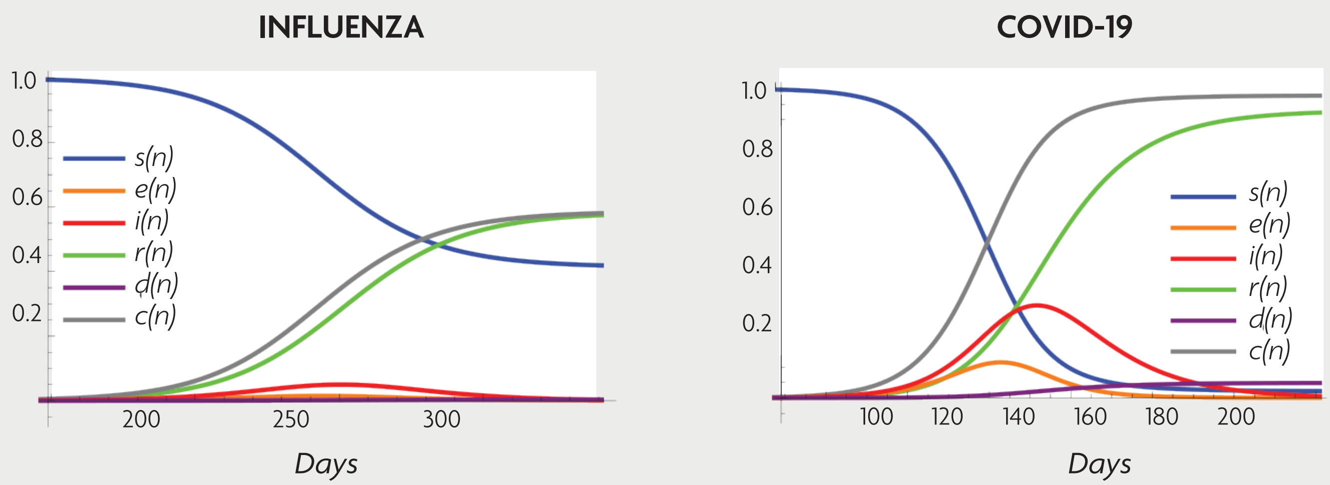
These parameters must be estimated from data on cases, hospitalizations, and deaths during the first weeks of an epidemic, along with some understanding of the transmission mechanisms of the disease. The first parameter, R0, is also directly informed by inputs of the average number of encounters per individual per day and how likely an encounter will result in an infection.

If the basic reproduction number is greater than 1, the outbreak is growing; if it is less than 1, the outbreak is fading away. The basic reproduction number or R0 for influenza is 1.5. For measles, which is highly infectious, R0 ranges from 12 to 18. The R0 of COVID-19 has been difficult to estimate, both because the large number of asymptomatic infections distort measurements and because physical distancing and other containment measures alter the number of people

SEIRD MODEL



INFECTIOUS DISEASE GROWTH SIMULATIONS



infected by a single individual. Nonetheless, the R0 for early COVID-19 variants was estimated to be at least 3.8, and thus COVID-19 is much more infectious than influenza. Since the R0 of COVID-19 is so high, any easing of preventive practices causes the epidemic to restart.

BEYOND THE CURVES

The variables *s(n)*, *e(n)*, *i(n)*, *r(n)*, and *d(n)* represent the proportions of susceptible, exposed, infected, recovered, and dead on day *n*. The graphs above depict growth simulations for influenza and COVID-19 without physical distancing measures. The gray curves represent cumulative cases, or *c(n)*, and it's these values that are typically shared by public sources.

The proportion of infected — depicted as the red curve — grows to a maximum and then decreases. In practical terms, as the proportion of the immune population grows, the likelihood of an individual encountering an infected individual declines significantly.

Absent physical distancing or other preventative measures, 30% of the population could be simultaneously infected at the COVID-19 peak. At this rate, the outbreak could easily overwhelm healthcare systems. By comparison, only 5% of the population could be simultaneously infected by influenza. Moreover, when the epidemics die out, 42% of the population remain unaffected by influenza, but only 2% for COVID-19.

Herd immunity, where a significant portion of the population is immune to an infectious disease, thereby conferring indirect protection even to those who are not immune, is a goal of most vaccination campaigns. However, it's only appropriate as a public health strategy when R0 is low and the illness is not severe. Otherwise, the risk of spread and preventable deaths is too high.

CONCLUSIONS

A single, simple yet robust mathematical model can highlight several characteristics of an epidemic that are essential to public health decision-making. The parameter values depend on the biological characteristics of the disease. To date, such models have been successfully applied to the SARS, Ebola, and mad cow epidemics at global and national scales.

For added precision, more refined SEIRD models can, for example, account for symptomatic and asymptomatic individuals or the possibility of an individual returning from the recovered compartment to the susceptible compartment.

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HARNESSING THE POWER OF DATA

ARTIFICIAL INTELLIGENCE-BASED PANDEMIC SUPPORT

Access to accurate, complete, and timely data is critical to understanding the characteristics of an epidemic. For the SARS-CoV-2 pandemic, securing consistent, high-quality data has been difficult, mainly due to the novel nature of the virus and the resurgent behavior of the outbreak. In Africa, these challenges are further exacerbated by resource limitations and reporting issues. A multi-disciplinary and multi-stakeholder team is using artificial intelligence techniques to identify patterns in COVID-19 data and produce mathematically-grounded, locally-nuanced analyses to inform science-based decision-making and design equitable and effective public health strategies across nine African countries.

KEY MESSAGES

- ☑ Artificial intelligence technologies and tools can play a key role in pandemic response and public health decision-making.
- ☑ In Africa, researchers have successfully delivered AI-powered, timely, and locally-nuanced analyses to monitor COVID-19, predict resurgences, isolate hot spots and outbreaks, identify individuals at higher risk of infection, stratify patients, identify gendered vulnerability, and devise effective and equitable public health measures and vaccination strategies.
- ☑ Multi-disciplinary and multi-stakeholder cooperation and data exchange, nationally and internationally, is critical to ensure that African countries — and the world as a whole — are better prepared for and more able to respond to future outbreaks.

Real-time delivery of credible information is critical throughout an epidemic to predict changes in the outbreak as early as possible, and guide public health measures. However, too little data, collected too slowly, can affect the accuracy of these predictions and the efficacy of mitigation efforts.

Advances in artificial intelligence (AI) can fill gaps in data and improve pandemic response at every stage. AI-powered tools have enabled monitoring the spread of COVID-19 at local, state, and national levels; predicting coming peaks and their intensities; identifying hot-spots; guiding purchase and allocation of health care resources; informing decisions and policies, both for closing down facilities and for reopening them; and optimizing vaccination roll-out strategies. These examples, along with other AI applications, have been successfully deployed in Africa to support the efforts of decision-makers, the medical community, and the public to manage every stage of the COVID-19 crisis.

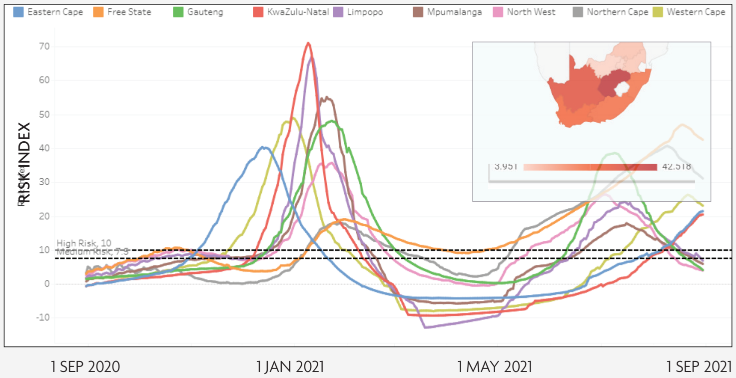
COLLABORATIVE INNOVATION

The Africa-Canada Artificial Intelligence and Data Innovation Consortium (ACADIC) brings together a multi-disciplinary and multi-stakeholder team of scientists, mathematicians, public health officials, and community leaders to exchange data and develop novel mathematical, artificial intelligence,

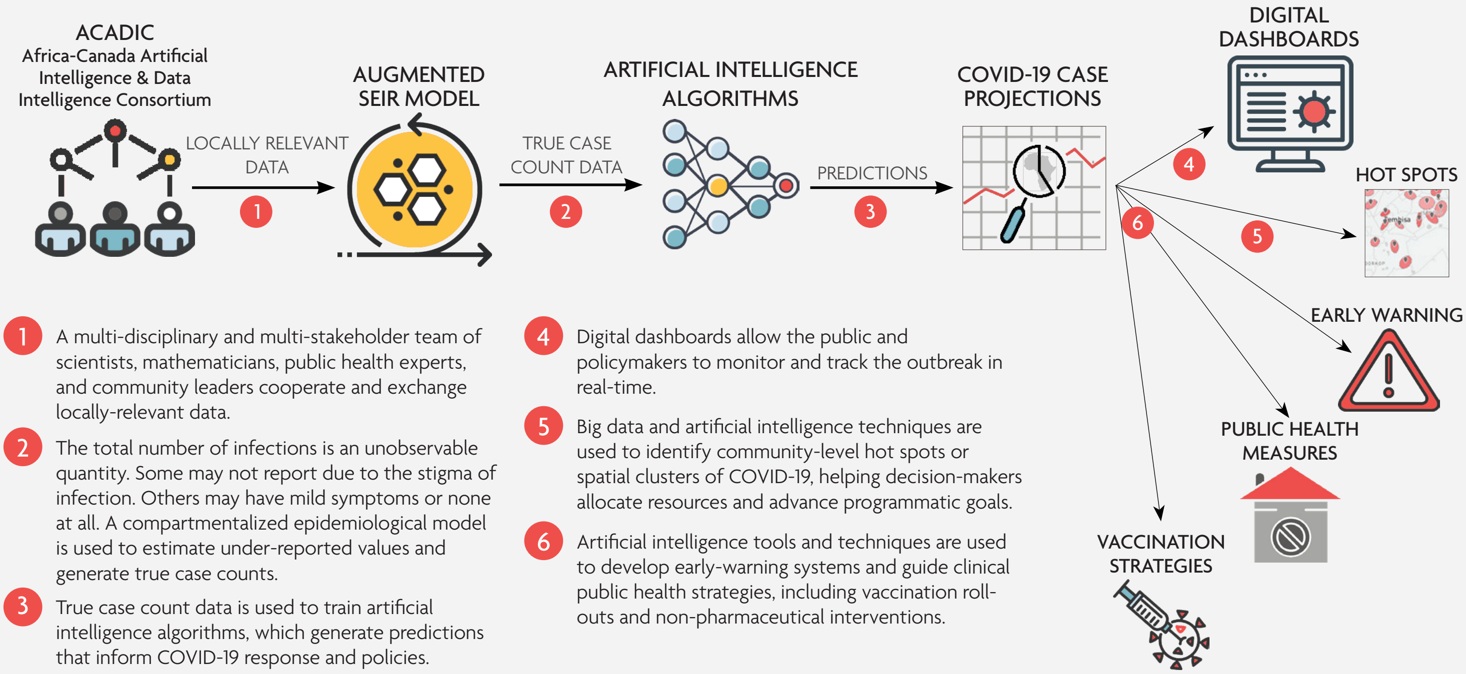
AN EARLY WARNING SYSTEM

Given the uneven and resurgent nature of the COVID-19 outbreak, an early-alert system to identify future waves is critically important to public health decision-making. Using mathematical models and AI algorithms, ACADIC researchers developed an alert system that measures the risk of potential future waves using risk indices. AI is used to uncover the complex interdependencies and relationships between human mobility patterns, the stringency of non-pharmaceutical interventions, and actual positive case data. These and other model outputs can be visualized in the COVID-19 South Africa Dashboard, which provides real-time, interactive virus monitoring for the public and policymakers.

3RD WAVE RISK INDEX BY SOUTH AFRICAN PROVINCE



SUPPORTING DATA-INFORMED DECISION-MAKING IN AFRICA



and statistical models to monitor the spread of SARS-CoV-2 and other infectious diseases.

The Consortium provides health care agencies in nine African countries with locally-nuanced analyses of data to help them make the best local public health decisions and policies.

By identifying and analyzing COVID-19 hot spots, outbreaks, and future waves, the group helps decision-makers proactively implement non-pharmaceutical public health measures to keep cases from overwhelming hospital capacity.

Using multi-dimensional data from local health authorities, the team trained artificial intelligence neural networks to identify vulnerable, high-risk groups based on risk. These indices help policymakers design equitable and effective vaccination roll-out strategies that maximize the impacts of available vaccines and ensure vulnerable communities are not missed. Equitable roll-out strategies are particularly important as most African nations have received very few vaccines.

Several regional consortia like the one described in this brief have been formed to help their respective policymakers monitor COVID. All five groups — three in Africa and two in Latin America — collaborate.

CONCLUSIONS

Big data and artificial intelligence have been, and are, continuing to play significant roles in the SARS-CoV-2 pandemic. In Africa, these tools and techniques can provide policymakers with a more accurate, timely, and locally-nuanced analysis to inform effective and equitable public health decision-making. Integrating the expertise and insights from multiple disciplines and countries with data and support from local officials is critical to ensure that African countries — and the world as a whole — are better prepared for and more able to respond to future outbreaks.

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IMPROVING PANDEMIC FORECASTS

ASSIMILATING OBSERVATIONS AND SIMULATIONS

At the height of the COVID-19 pandemic, an international team of mathematicians borrowed techniques from the geosciences to predict the complex and shifting dynamics of the virus' spread. The technique known as data assimilation combines numerical model data with fresh observational data to deliver more accurate forecasts. Validating the method in eight distinct countries, the team demonstrated the potential to reasonably and accurately predict the short-term impacts of various reopening measures on virus transmission. This method can provide critical information to policymakers to make informed decisions and design effective policies to mitigate the pandemic's impacts.

KEY MESSAGES

- ☑ An international team of mathematicians employed geosciences-derived data assimilation methods to enhance the prediction accuracy of traditional epidemiological models.
- ☑ The team introduced a variant of the susceptible-exposed-infected-recovered, SEIR, model, a standard mathematical approach to forecast infectious disease transmission. By combining the model information with new observational data for death and hospitalization numbers, they were able to produce realistic predictions of the pandemic's evolution, with quantified uncertainty estimates, for eight distinct countries.
- ☑ Mathematical data assimilation methods can play an important role in predicting a pandemic's evolution.
- ☑ The method can provide real-time, data-informed forecasts to policymakers to implement effective interventions to control a pandemic's spread and mitigate its impacts.

COMBINING THE BEST OF BOTH

As scientists scrambled to understand SARS-CoV-2, decision-makers and the public reasonably asked: How many more people are likely to die, and what effect will governmental containment policies have on the virus' spread? To answer these questions, scientists turn to numerical models, which describe the relationships between epidemic parameters or variables, and observational data, such as the number of individuals who died or were hospitalized due to the virus.

Neither numerical models nor observational data alone can accurately answer such questions, but by objectively combining the two, scientists can utilize the best parts of each while minimizing their respective flaws. The technique, known as data assimilation, is used routinely in geosciences, for example, in modern-day weather forecasting, arguably the best known and most successful application. With improved numerical models and data assimilation, today's 5-day weather forecast is as accurate as a 1-day forecast was in 1980.

In addition to improved forecast accuracy, data assimilation provides a robust assessment of the uncertainties of the output, a significant benefit over simpler, free-running models. In this regard, it offers predictions of the worst, best, and most likely situations.

AN INTERNATIONAL INITIATIVE

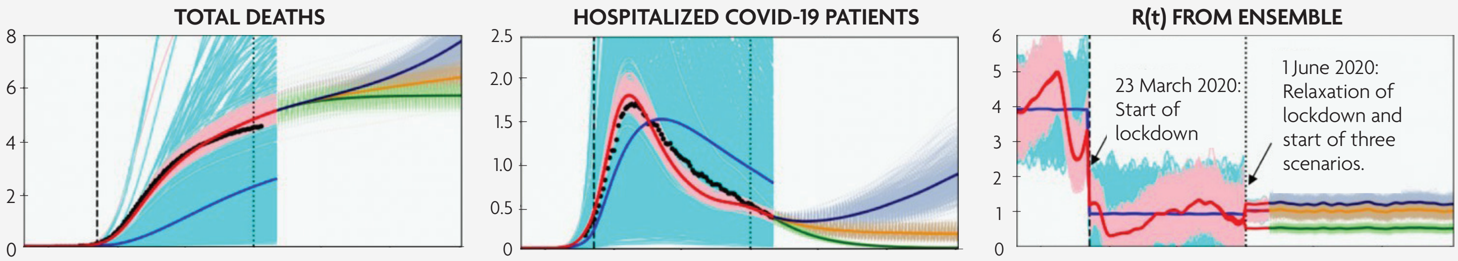
In the spring of 2020, an international team of data assimilation scientists representing eight countries diverted their attention from their work in Earth science fields to explore whether data assimilation tools could be applied to pandemic modeling in each of their countries. As the evolution of the epidemic varied widely between countries. Hence, the team began by identifying the factors that potentially contributed to virus transmission, such as geography, population density, social habits, healthcare systems, and, importantly, governmental policies and mitigation strategies, including lockdowns.

The team found that they could use data assimilation to explain reported deaths and hospitalizations using a classic metapopulation model, a type of spatial model that explores interactions of subpopulations across time and space. Their model is a version of the susceptible-exposed-infected-recovered (SEIR) compartment model, adapted to COVID-19 by including age-stratification and additional compartments for quarantined, hospitalized, and deceased.

Using this approach, they successfully represented the impact of the various interventions taken in each of their eight countries: Visualizing the rapid drop-off in person-to-person transmission at different points in each country's lockdown. Given the success of data assimilation to explain the reported deaths, they went on to develop predictions

CASE STUDY: EVOLUTION OF THE COVID-19 EPIDEMIC IN ENGLAND

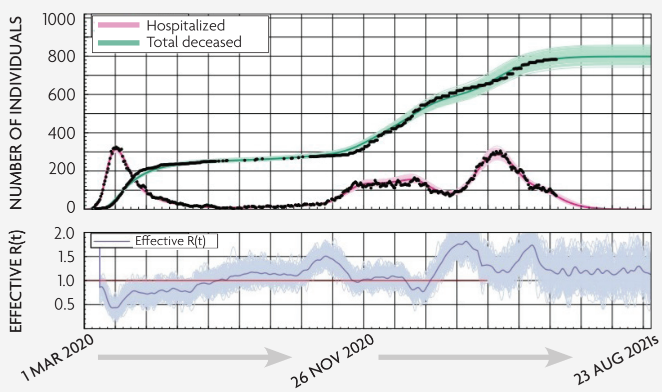
For England, simulations start on February 20th and assimilate data on a daily cycle beginning March 5th. Daily data for total deaths are assimilated from March 5th to May 29th, together with daily hospitalization numbers from March 20th to May 29th. Over the assimilation period, data are assimilated sequentially as they become available at a daily rate. The final assimilation solution over the whole data period is thus informed on all observations. Data after May 29th are not used in the assimilation, and the simulations run unconstrained. Three possible scenarios were considered beginning June 1st of 2020 when lockdown restrictions started to ease. Scenarios were defined in terms of the R naught value, which represents the number of people an infected person can be expected to transmit the virus to. They chose three values: 1) R equals 0.5, where cases reduce over time; 2) R equals 1.0, where case numbers remain steady over time; and 3) R equals 1.2, where case numbers increase over time. On June 1, approximately 45,000 deaths in England were attributed to COVID-19 in all settings. The team's analyses under the three scenarios projected that by the 1st of September 2020, total deaths would equal 57,000 00 ﬄ 1,900 (R=0.5), 63,600 ﬄ 2,700 (R=1), and 76,400 ﬄ 4,900 (R=1.2). These results highlight the potential to save tens of thousands of lives by using containment measures that reduce a significant amount of person-to-person contact.



The black dots depict reported values up to June 5th for deaths and up to June 12th for the number hospitalized. The bright blue lines indicate the initial estimates, and the red lines indicate the values after assimilation, with the bold line indicating the most likely value. After the 1st of June, three predictions are made based on three different R values: R=1.2 (navy), R=1 (yellow), and R=0.5 (green).

CASE STUDY: EFFECTS OF THE VACCINATION CAMPAIGN IN NORWAY

Using more sophisticated data assimilation methods, the team modeled the predicted effect of Norway's vaccination campaign. This new version of the model illustrates the power to forecast the pandemic's evolution over a longer period. In this case, the predictions' uncertainties reflect the uncertainties in both the simple model and the reported values for deaths, hospitalizations, and positive cases.



Top: hospitalized and total dead with observations (black dots). Bottom: Reproductive number R. In both panels shading shows the estimated uncertainties.

under different possible scenarios, such as reopening strategies and vaccination campaigns.

CONCLUSIONS

A popular framework in geosciences, numerical and mathematical data assimilation methods have proven extraordinarily versatile and provide a dynamical and statistically-sound way to combine multiple pandemic measurements with numerical models of its evolution. By quantifying model and observation uncertainties and including new epidemic data, the team provided reliable short-term predictions of pandemic indicators — deaths, infections, and hospitalizations — and estimates of the accuracy of these numbers. Across eight different countries and states, the team demonstrated the method's capability to detect the impact of each region's governmental interventions and assess their effects on the SARS-CoV-2 pandemic evolution.

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ENHANCING VACCINE DESIGN

MATHEMATICS ACCELERATES MEDICAL INNOVATIONS

Mathematical tools are increasingly being used to provide insights into a range of public health issues, including in the areas of vaccine discovery, development, and testing. Combined with emerging technologies, mathematical approaches help scientists decode the genetic sequences of pathogens. Sophisticated algorithms are then used to identify which parts of the pathogen will be recognized by a body's immune system. Mathematical and statistical models also underpin clinical trial design and data analysis. By accelerating and enhancing the design of effective new vaccines and enabling the redesign of existing ones, mathematics reduces the threat of epidemics and improves global health.

Vaccines are among the most significant public health achievements of modern times. Before vaccine innovations, outbreaks of infectious diseases had profound and lasting effects on global economies and development and, in some cases, decimated populations. Between 1346 and 1353, the Black Death plague pandemic raged across the Middle East, North Africa, and Europe, killing an estimated 40% to 60% of the population. The Spanish flu, an influenza pandemic, spread globally from 1918 to 1919, killing an estimated 50 million people. And as of 2021, the SARS CoV-2 pandemic has claimed over 5 million lives, though experts suggest that number is likely much higher.

KEY MESSAGES

- ☑ Vaccines have helped save millions of lives, reduced healthcare costs, and elevated the quality of human and animal lives. They contribute to social and economic development and are vital to achieving the Sustainable Development Goals.
- ☑ Mathematical modeling supports efficient vaccine development by capturing complex immunological dynamics and predicting which parts of a pathogen will most likely induce an immunogenic response.
- ☑ Mathematical and statistical models can help design clinical trials and vaccine effectiveness studies, and analyze their results, advancing safer and more efficacious vaccines.
- ☑ Sophisticated mathematical algorithms can rapidly decipher pathogen genome sequences and predict or track changes to these sequences that may reduce vaccine effectiveness. These analyses are used to select new vaccine designs.

THE IMMUNOLOGY OF VACCINATION

An organism protects itself from pathogens through defense mechanisms that physically prevent viruses or bacteria from entering its body or identify and destroy them if they do.

The human immune system uses two main strategies. Upon exposure to a previously unseen pathogen, the body counters first with an innate, generalist immune response, affording the host time to mount a larger, more specific response. The second-line adaptive immune response results in the production of antigen-specific antibodies. Once the adaptive immune system is primed, it's able to quickly and effectively defend against the same pathogen if it ever invades again. Vaccines utilize this second-line response by exposing the body to a priming agent that elicits an immune reaction, training the adaptive immune system to recognize and attack pathogens without causing disease.

MEASURING PROTECTION

Demonstrating how well vaccines work is fundamental to helping inform policy- and decision-makers about vaccines' potential use and value. Statistical methods support the design of clinical trials and the analysis of data.

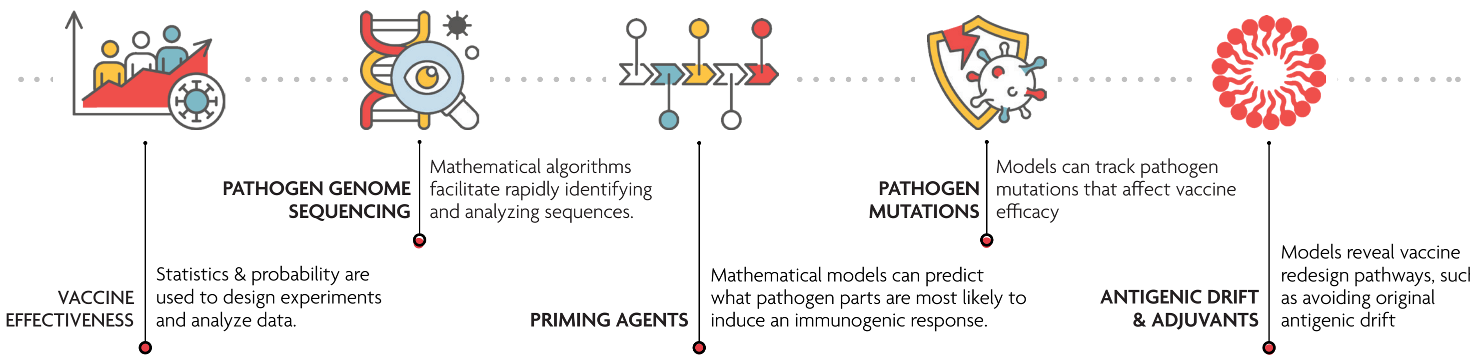
Vaccine effectiveness is generally measured by calculating the risk of disease among vaccinated and unvaccinated individuals and determining the percentage reduction in risk of disease among vaccinated individuals relative to unvaccinated individuals. The basic mathematical equation is written as:

$$\mathbf{VE} = \frac{\text{Incidence Among Unvaccinated Group - Incidence Among Vaccinated Group}}{\text{Incidence Among Unvaccinated Group}}$$

Expressed as a percentage

The greater the percentage reduction of target infection in the vaccinated group, the greater the vaccine effectiveness.

MATHEMATICAL APPROACHES GUIDE VACCINE DESIGN & DEVELOPMENT



COMPUTATIONAL IMMUNOLOGY

Mathematical approaches facilitate rapidly identifying and analyzing pathogen genome sequences, enabling the efficient development and effective design of vaccines. The first genetic sequences of the SARS-CoV-2 virus were derived using nanopore sequencing, a technique in which single strands of DNA or RNA molecules are passed through a tiny channel — a nanopore — embedded in an electrico-resistant membrane. As the molecule passes through the nanopore, it causes disruptions in the electrical current. The raw disruption signal data is then converted into a sequence using sophisticated mathematical algorithms. Each nucleotide base that passes through the nanopore can be identified through this characteristic disruption sequence in real-time.

Mathematical models use this genetic sequence data to predict those parts of the pathogen that are suitable priming agents, reducing the time and cost of more expensive methods. Recent technological advances make it possible to deliver these priming agents using the molecular form of messenger RNA, or mRNA. Improvements in mRNA vaccines have allowed COVID-19 vaccines to be developed at record speed.

TRACKING DRIFT

Pathogens mutate to evade natural or vaccine-elicited immunity. In many cases, these acquired genetic changes give the pathogen the means to overcome the body's vaccine-primed immune response, necessitating vaccine redesign to restore its effectiveness.

Mathematical models are used to build phylogenetic trees that illustrate how the genetic sequences of circulating viruses are related to that of the current vaccine virus. Models are also used to predict the extent to which such genetic changes reduce vaccine effectiveness and inform the selection of new vaccine designs. The flu vaccine, for example, must be reviewed and updated each year to keep pace with the influenza virus as it mutates quickly. Similar strategies are being used to monitor changes to the SARS-CoV-2 virus to determine the extent to which newly emerged variants undermine the effectiveness of existing COVID-19 vaccines.

AVOIDING ORIGINAL SIN

Studies have demonstrated that redesigned vaccines may preferentially prime the immune system against the original pathogen strain and provide only suboptimal immunity against the new targeted variants. Mathematical models reveal that this phenomenon — known as the 'original antigenic sin' — can be alleviated if the priming conditions involve adjuvants that strongly activate first-line immune defense systems. This revelation offers a pathway for redesigning vaccines while preventing original antigenic sin from undermining vaccine effectiveness. mRNA vaccines tend to activate the first-line defense systems better than other vaccination modalities, suggesting these vaccines might be less affected by the original antigenic sin.

CONCLUSIONS

Vaccines are one of the most cost-effective ways to save lives. Mathematical and statistical approaches have helped to accelerate and enhance vaccine design, development, testing, and delivery; have made vaccines safer and more effective; and, in doing so, have transformed lives.

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MODELING VACCINE HESITANCY

UNDERSTANDING THE FREE RIDERS OF HERD IMMUNITY

Vaccines are unquestionably one of the greatest medical advances in modern times, saving millions of lives worldwide every year. Since the 20th century, ten historically fatal diseases have been nearly or fully eradicated. Smallpox was globally eradicated from the Earth by 1980, measles had been eliminated in many countries by the 1990s, and polio is nearly gone. But an uptick in vaccine hesitancy now threatens to derail such vaccination campaigns. Mathematics can help provide insights into fundamental interactions between disease spread and vaccinating behavior. These insights can improve public health strategies to locally eliminate and globally eradicate infectious diseases.

KEY MESSAGES

- ☑ Vaccine hesitancy is a global public health challenge, affecting countries at all levels of development.
- ☑ By synthesizing the economic and psychological aspects of human behavior with epidemiological dynamics, mathematical models can provide new insights that help to explain human decision-making with respect to vaccination.
- ☑ Game-theoretical models often forecast a rise in vaccine hesitancy as infections decline. As the population approaches the herd immunity threshold, any perceived vaccine risk will eventually outweigh the progressively smaller risk of infection. As a result, self-interest might preclude complete eradication of vaccine-preventable diseases.
- ☑ The design of local elimination and global eradication programs for vaccine-preventable infections should both anticipate — and offer strategies to counter — vaccine hesitancy close to the herd immunity threshold.

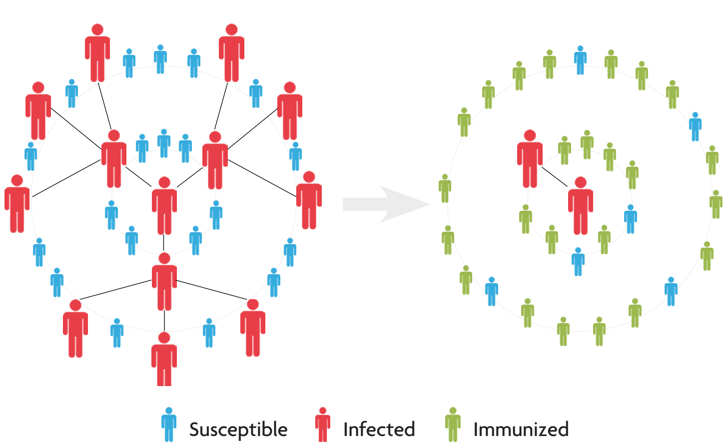
PROTECTING THE HERD

An important goal of many vaccination campaigns is to increase population immunity. To illustrate the power of vaccines, imagine a scenario where the average person typically has ten close contacts a day. Suppose they become infected with a disease that causes them to pass on the disease to three other people, on average. Those three will each infect another three people, who will themselves pass it on, and so on, resulting in an exponentially growing epidemic. In this scenario, three is the basic reproductive number or R0 of the infection — the average number of new infections caused by a single infected case in a susceptible population.

VISUALIZING THE EFFECT OF VACCINE CAMPAIGNS

NO VACCINE (R0=3)
Exponentially Growing Outbreak

80% VACCINATED (R0<1)
Outbreak Dies Out



Now assume that 80% of the population has been vaccinated and cannot become infected or pass the infection to others. The number of infections caused by the initial case is greatly reduced, down to zero or perhaps just one. Even if those infected do pass on the disease to their unvaccinated contacts, that new case of infection would, in turn, only cause zero or one additional case. Eventually, the disease would die out as the few people who aren't immune are less likely to come into contact with someone infected. The proportion of immune people needed to achieve this point of "herd immunity" is approximated by a simple mathematical formula:

$$p = 1 - 1/R0$$

GETTING THE PRISONERS IN THIS DILEMMA TO COOPERATE

Mathematical game theory has long-standing traditions in economic theory. At its core, it models competitive and cooperative human behavior, where each individual has choices, but the payoff for each choice depends on choices made by others. A classic example is the prisoner's dilemma game, where two suspects are arrested and separated by the police. Each has to decide whether to confess to the crime without knowing what the other will do. The payoff for each depends on the choice made by the other.

A \ B	Suspect B Stays Silent	Suspect B Betrays
Suspect A Stays Silent	B: 2 years / A: 2 years	B: 1 year / A: 4 years
Suspect A Betrays	B: 4 years / A: 1 year	B: 3 years / A: 3 years

Suspect A thinks: "If suspect B decides to betray me, I should also betray them to get three years in prison, instead of staying silent and getting four years. And if suspect B decides to remain silent, I should still betray them to get only one year in prison instead of staying silent and getting two years. Either way, it makes sense to betray." Suspect B is thinking the same thing. As a result, they each spend three years in prison, even though they would only have spent two years in prison if they both remained silent.

These situations, where the outcome is less optimal for the common good, exemplify the free-rider problem in vaccination. In economic theory, free riders benefit from public goods but do not pay for them or underpay. In the context of an epidemic, free riders are those who enjoy the benefits when others get vaccinated — lower disease transmission and eventual herd immunity — but refuse to accept the small cost of getting vaccinated themselves.

Game theory models are not intended to perfectly represent reality. Vaccine hesitant individuals are a diverse group with diverse motivations. Only a few strongly anti-vaccine individuals may never be convinced, while many more are willing to get the vaccine under the right circumstances. Similarly, vaccines are not all the same — some prevent disease but do not block transmission. Moreover, populations are subject to effects like social norms and social learning that can ultimately support vaccination. Simplifications aside, game theory illuminates an underlying factor that causes more vaccine hesitancy to occur closer to the herd immunity threshold. And a game theory approach does not require assuming that individuals think like game theorists, only that they notice a drop in infections and perceive a reduced infection risk when vaccine rates increase.

Mathematicians are actively developing more sophisticated models that capture realistic features, such as social learning. These have been shown to explain childhood vaccination behavior data better than models that ignore evolutionary game theory. They're also applying similar frameworks to a wider range of issues, such as understanding the interactions between climate change impacts and public opinion about climate mitigation. These coupled socio-climate models show that social processes can dramatically alter the projected peak global temperature.

CONCLUSIONS

Mathematical tools offer the power and flexibility to describe, investigate, and solve problems across a range of complex issues, such as the game-theoretical aspects of both climate change and vaccine uptake. Cross-disciplinary game-theoretical models can provide powerful insights for public health decision-making. As populations approach herd immunity, policymakers should anticipate a rise in vaccine hesitancy as infection rates decline. Global eradication and local elimination programs could anticipate such an eventuality and develop the means to address it. Vaccination campaigns should focus on addressing the logistical challenges in vaccinating hard-to-reach populations and building communication and trust with those vaccine-hesitant individuals who are willing to listen.

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TEACHING MATHEMATICS

MATHEMATICS EDUCATION FOR SUSTAINABLE DEVELOPMENT

Providing equitable access to quality education for all children is an urgent goal for global sustainable development. Mathematics matters because it permeates all aspects of daily life, and learning mathematics develops 21st-century skills related to critical thinking and problem-solving. While it can be challenging to disentangle the myriad social, economic, and institutional factors that influence outcomes for learners, there is clear evidence that teachers are the key influence on student learning at the classroom and school level. The quality of mathematics teachers is especially significant since mathematics achievement has a profound impact on young people's life chances after leaving school.

Equitable access to quality education positively impacts a wide range of sustainable development outcomes, such as reducing intergenerational poverty and promoting better health, cultural diversity, and gender equality.

It's especially important to provide all children with access to high-quality mathematics education. Students who leave school with poor achievement in fundamental mathematics are more likely to experience unemployment or low-paid work, poor physical and mental health, and low levels of civic participation. Mathematics also provides the essential foundation for critical citizenship and lifelong learning in a world characterized by rapid technological, social, and economic change. The success of a mathematics education is measured through the abilities of learners to transfer their knowledge and problem-solving skills to new situations. Teachers are the key to achieving these goals.

> **The quality of an education system can exceed neither the quality of its teachers nor the quality of its teaching.**
>
> — UNESCO

THE PROBLEM OF MATHEMATICS TEACHER SUPPLY

However, there is a global shortage of mathematics teachers that creates challenges for the provision of quality mathematics education.

KEY MESSAGES

- ☑ Mathematics education develops problem-solving and critical-thinking skills that can be transferred to new situations and a range of occupational fields.
- ☑ Mathematics education is important for developing reflective and critical citizens who can deal with the mathematical demands of everyday life, and also for preparing a sufficient number of mathematicians and scientists capable of meeting the challenges of the contemporary world.
- ☑ Quality mathematics teachers are the key to improving young people's learning outcomes and socioeconomic mobility. Governments should address the global shortage of quality mathematics teachers which threatens achievement of these goals.
- ☑ Quality in mathematics teaching is not the same as having advanced mathematics or education qualifications. A quality mathematics teacher needs to have mathematical knowledge for teaching.

There are several reasons for the problem of mathematics teacher supply. The rapid expansion of school education systems in low or middle-income countries, perception of teaching as a low-status profession, lack of attractive career paths, and active competition from emerging professions -- for example, banking, computing, data science -- have led to teacher shortages in many countries. There are additional challenges in ensuring the supply of quality mathematics teachers in contexts of disadvantage and diversity.

Programs are offered to address teacher shortages in many countries. For example, these include the recruitment of para-teachers in India or the Teach for America program. However, these programs, possibly inadvertently, do not adequately prepare the teachers for classroom teaching in mathematics. Increasing the supply of quality mathematics teachers is not simply a matter of attaining qualifications in advanced mathematics or completing an education degree.

WHAT IS MEANT BY A "QUALITY" MATHEMATICS TEACHER?

Neither conventional mathematics content knowledge nor knowledge of general pedagogical strategies is enough for managing the mathematical teaching tasks. Research has shown that there is little correlation between students' achievement gains in mathematics and indicators of teacher qualifications, such as the number of mathematics courses teachers have taken. Instead, teachers need to have mathematical knowledge for teaching. This is a form of professional knowledge that blends knowledge of mathematics, how to represent mathematics so that it can be understood by learners, and how to respond to students' emerging and incomplete thinking in mathematics.

Teachers must also possess knowledge "at the mathematical horizon," that is, awareness of connections to more advanced mathematical ideas than the content they teach, for example, advanced mathematical modeling. Because the horizon keeps shifting as the world changes and becomes more complex, mathematics teachers need to constantly update and expand their professional knowledge in all areas of mathematics.

DEVELOPING QUALITY MATHEMATICS TEACHERS

Quality teachers need to acquire mathematical knowledge for teaching, an intricate blend of mathematics content knowledge and pedagogical content knowledge. These two forms of knowledge can be integrated within teaching practice if mathematicians and mathematics educators work together in teacher education and development projects so that there is mutual reinforcement of professional and academic knowledge.

The Capacity and Network Project of the International Commission on Mathematical Instruction is an example of a potentially sustainable model for interdisciplinary collaboration. The Network brings together mathematicians and mathematics educators to help teachers in low and middle-income countries develop their mathematical knowledge for teaching. Another example of such collaboration is the Klein project, which produces mathematics resources for secondary school teachers that make connections between the mathematics they teach and advanced topics in contemporary mathematics.

CONCLUSIONS

Education empowers learners, helps reduce inequalities, enables upward socioeconomic mobility, and fosters more tolerant and just societies. Improving access to quality education is a global imperative that requires substantial investment by education systems. Recruiting and developing quality teachers through pre-service and in-service education is central to achieving this goal. Mathematics education can provide research-informed evidence on what it means to be a "quality" mathematics teacher and how to develop the mathematical knowledge for teaching that is the hallmark of this quality. Mathematicians and mathematics educators can collaborate to deliver programs that develop teachers' mathematical knowledge for teaching. These interdisciplinary networks are a potentially sustainable model for teacher development in low- and middle-income countries.

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TRACKING GENDER PARITY

MATHEMATICAL FOUNDATIONS OF GENDER EQUALITY INDICATORS

Gender equality is a prerequisite for sustainable development and the alleviation of poverty. Empowered women and girls contribute to the health and productivity of their families, communities, and countries. In the past few decades, global institutions have developed a range of indicators to measure and track the legal, economic, social, and cultural factors contributing to a gender gap. These composite indicators simplify complex information by mathematically aggregating individual measures into a single summary indicator. The usefulness of these composite indicators depends heavily on the underlying weighting and aggregation methods.

KEY MESSAGES

- ☑ Composite indicators are a mathematical aggregation of a set of individual indicators that measure multi-dimensional concepts, but usually have no common unit of measurement, They're increasingly used by global institutions to enhance public debate, benchmark performance, and analyze policies.
- ☑ Multiple composite indicators are used to measure and track the legal, economic, social, and cultural drivers of the gender gap.
- ☑ The proliferation of composite indicators for decision- and policymaking raises issues about accuracy, robustness, and reliability. Poor composite indicators can lead to poor decisions and ineffective policies.
- ☑ Mathematical approaches provide robust means to analyze the sensitivity of composite indicators and uncover which dimensions contribute most to closing the gender gap, providing the means to measure the gender gap in more accurate and reliable ways.

As codified in the 1948 Universal Declaration of Human Rights and adopted as a treaty by the United Nations General Assembly in 1979, gender equality is a fundamental human right and an essential foundation of a sustainable, peaceful, and prosperous world. However, while there has been some progress in closing the gender gap, there is still a long way to go before women and men enjoy the same rights and opportunities across all aspects of life, especially economic participation and political power.

Information on changes in gender parity is of political significance since the United Nations member states agreed to advance gender equality and the empowerment of women and girls as part of the 2030 Agenda for Sustainable Development. Such information is often provided through composite indicators, whose purpose is to simplify complex information and provide a comprehensible framework to measure, communicate, and track progress towards meeting policy goals and targets over time.

MEASURING COMPLEX PHENOMENA

Composite indicators mathematically aggregate a set of individual indicators, usually with no common unit of measurement, into a single summary indicator. Ideally, composite indicators are based on a statistical and mathematical framework that selects, combines, and weights the individual indicators to best reflect the dimensions or structure of the measured phenomena. To a large extent, the usefulness and reliability of a composite indicator depend on the underlying weighting, scaling, and aggregation methods.

For example, if the single indicators use different units of measurement or scale, small variations in larger-scale indicators contribute to more significant changes in the composite indicator score. Mathematicians and statisticians have provided guidance on averaging procedures and normalization methods that allow for meaningful composite indicator values.

Composite indicators are a widely used and popular tool for performance monitoring, benchmarking, policy analysis, and communication with decision-makers and the general public on key policy issues, such as health, the environment, and sustainability. Well-known composite indicators include the Human Development Index, the Environmental Performance Index, the Social Progress Index, and the Global Innovation Index.

DATA & GENDER EQUALITY INDICES

A number of composite indicators attempt to measure the state of gender equality, among these:

VALIDATING PROGRESS: CLOSING THE GENDER GAP

GGGI measures four equally-weighted dimensions: Economic participation & opportunity, educational attainment, health & survival, and political empowerment. Country rankings and index scores vary considerably depending on the choice of dimensional weights. World Economic Forum statisticians chose to weight these dimensions equally, but with a different choice of weights, the indicator scores would be quite different.

GGGI Weighting Example: Algeria & Angola

With equal weighting, the 2021 composite score for Algeria is 0.633, and for Angola, it's 0.657. If stakeholders are particularly interested in educational parity but also value health and survival, they might weight education at 50%, health at 30%, and the other dimensions at 10%. In such a case, Algeria's weighted average moves up to 0.831, and Angola moves up to 0.762. Both would have a higher GGGI score, and Algeria would now surpass Angola on gender parity. Suppose economic participation is weighted at 50%, health at 30%, and the other dimensions at 10%, In that case, Algeria's weighted average drops to 0.627, Angola's average increases to 0.717, and Angola once again surpasses Algeria on gender parity. Therefore weights are crucial and should be selected to represent the interests of stakeholders best.

Country	Algeria	Angola
Economics	0.456	0.646
Education	0.966	0.759
Health	0.958	0.979
Political	0.151	0.245
Equally-weighted Average	0.633	**0.657**
Weighted to Education (50%), Health (30%), Others (10%)	**0.831**	0.762
Weighted to Economic Participation (50%), Health (30%), Others (10%)	0.627	**0.717**

- **The Global Gender Gap Index (GGGI) —** The World Economic Forum first introduced the GGGI in 2006 as a framework for capturing gender-based disparities and tracking their progress over time. The GGGI benchmarks national gender gaps on economic, education, health, and political criteria and provides country rankings that allow for comparisons across regions and income groups.
- **Gender Inequality Index (GII) —** The United Nations Development Programme's GII is a composite measure of gender-based disadvantage in three dimensions: Reproductive health, female empowerment, and labor market participation. A higher GII value equates to more disparities between females and males and a greater loss to human development.
- **Social Institutions and Gender Index (SIGI) —** The SIGI, compiled by the Organization for Economic Cooperation and Development, measures discrimination against women in social institutions. The SIGI is an unweighted composite index comprised of four sub-indices: Discrimination in the family, restricted physical integrity, restricted access to productive and financial resources, and restricted civil liberties. A SIGI value of 0 indicates complete equality; a value of 1 indicates complete inequality. The SIGI is one of the official data sources for monitoring SDG 5.1.1.

CONCLUSIONS

Despite their popularity, composite indicators have met with some criticism, particularly around the steps of weighting and aggregation and the statistical significance of the final product. To improve the accuracy and reliability of composite indicators, mathematicians are continuing to propose new, robust approaches to their construction, including the use of interval values to measure composite indicator uncertainty based on the different assumptions used as inputs to evaluate the implicit weights of the dimensions used in the measure of the composite indicator.

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MANAGING WATER RESOURCES

PROBABILISTIC TOOLS FOR WATER RESOURCES VULNERABILITY

Access to reliable and secure drinking water is essential for health, agriculture, sanitation, and hygiene. The World Health Organization promotes water safety through a series of water quality guidelines based on identifying and managing risks from catchment to consumer. For water suppliers and decision-makers, Bayes' Theorem offers a useful mathematical tool for quantifying risks and identifying appropriate options for the management of water supply and quality. Bayesian approaches can lead to probability estimates in the face of uncertainty that support better-informed decisions in government and public policy. These methods are now widely used in various fields, including medicine, law, and ecology..

KEY MESSAGES

- ☑ A mathematical method developed by an 18th-century Presbyterian minister has proven especially useful in helping government, public utilities, and the private sector approach complex problems involving water supply and water quality. The method, known as Bayes' Theorem, allows for estimating the likelihood of an event based on prior information or knowledge.
- ☑ Access to reliable and secure drinking water is essential for health and sustainable development. Bayes' Theorem holds considerable promise as a decision support tool for evaluating and prioritizing management and maintenance of water resources and water quality.
- ☑ Bayesian approaches are used to identify robust ways of quantifying risk and protecting human and environmental health.. Their use can lead to probability estimates that support better-informed decisions in government and public policy, in particular for water supply.

WATER RELIABILITY

Short-term water supply disruptions can be inconvenient, but a longer-term outage can threaten human health or life, especially if it occurs in the context of an epidemiological emergency, earthquake, or other natural hazards, or at a location where alternative sources of water are in short supply, expensive, or difficult to obtain. Water shortages pose an especially significant threat in developing countries. For communities that rely on groundwater accessed through boreholes and pumps, failure prediction is critical to avoid disruptions in water access or supply.

QUANTIFYING RISK

Suppose a water supply company is looking to minimize the likelihood of service outages lasting more than three hours. They would need to determine the most common causes or failures that result in these longer outages.

Typically, characteristics of water supply outages are detailed by the company in a failure book. Using this data, the company can calculate the following values: a) the number of specific outages attributed to each kind of failure over a given period; b) the number of water supply outages assigned to each duration class, for example, <3 hours and ≥3 hours; and c) for each type of failure, the number of resulting outages assigned to each duration class.

If there is a failure such as a leaking pipe, water supply managers can use this data to determine the likelihood it will cause an outage of at least three hours. But in the future, they might want to know the opposite: If they see an outage of three or more hours, what is the most probable failure that led to it?

Problems such as these lend themselves to a surprisingly powerful tool developed in the 18th-century by an English Presbyterian minister named Thomas Bayes.

A LOOK AT BAYES' THEOREM

Reverend Bayes, a trained statistician, developed a method for estimating the probability of a future event based on observed evidence. Estimates can be regularly updated using this method as new evidence is gathered. Bayes' Theorem is stated mathematically as:

the **conditional** probability of the occurrence of an outage of class C given the occurrence of failure F

"a priori" probability of the occurrence of failure F

$$P(F|C) = \frac{P(C|F) \times P(F)}{P(C)}$$

"a posteriori" probability of the occurrence of failure F, given the occurrence of an outage of class C

probability of the occurrence of an outage of class C

Prior or *a priori* probabilities are the initial probability values. *A posteriori* probability is the probability value after new evidence is incorporated, for example when 'C' occurs. The *a posteriori* probability estimate can be regularly updated using Bayesian methods as new evidence is gathered.

BAYES' THEOREM IN ACTION

The outage data in the first two rows of the following table is from a Polish water supply company that wanted to determine the probability that a future outage of three or more hours would be caused by a specific type of failure:

Outages/Failure	Leaking Pipes	Damage to Water Fittings	Pipe Corrosion	Cracking Pipes	Other	Total
# Due to Failure (F)	94	88	81	57	35	355
# Outages ≥3 hrs (C)	50	20	49	27	23	169
P(F|C)	**0.2958**	0.1183	0.2899	0.1598	0.1361	

If F is a leaking pipe and C is an outage of at least three hours, Bayes' Theorem gives the *a posteriori* probability — **P(F|C)** — that if there is a future outage of at least three hours that it will be due to a leaking pipe. Using present data, or *a priori* probabilities, **P(F|C) = 0.2958**. The use of more data has given a more refined estimate than simply **P(F)**.

To reduce the incidence of water supply interruptions, companies can use these probabilities to inform strategic decisions, such as where to target inspections, when to repair or replace materials, and how to prioritize limited resources. And with this method, the probabilities can be regularly updated as new failure data is recorded.

In Kenya, the tool eMaji Manager uses sensor data to manage waterpoint access for people and livestock and minimize failures at boreholes in semi-arid regions in the northern part of the country. This tool and Bayesian methods related to machine learning can lead to more robust failure prediction.

MONITORING WATER POLLUTANTS

Because of its unique approach to quantifying uncertainty and variability, Bayes' Theorem has proven to be an effective tool for understanding water quality conditions. For example, because water pollutant concentrations typically vary by month, Bayes' Theorem can be used to determine the likelihood that water samples in violation of given water-quality standards will occur in a particular month.

Bayesian approaches like this have been used to guide management decisions and design regulatory measures related to water quality standards. For example, the City of Austin, Texas, utilized Bayesian methods to assess stream water quality in relation to wastewater treatment plant effluent. The State of North Carolina used Bayesian methods to evaluate river eutrophication for proposed nitrogen limits in the Neuse River. And Bayesian methods have been used to study the relationship between water flow and salinity in Australian rivers.

CONCLUSIONS

Analysis using Bayes' Theorem has proven helpful across a variety of applications, including epidemiology, environmental policy, medical decision-making, and legal proceedings. Bayesian approaches are constructive in uncertain situations that call for probabilistic thinking, where estimates need to be updated regularly as new evidence or information is encountered. In this context, Bayesian approaches can lead to more accurate probability estimates and thus better-informed decisions in the face of uncertainty.

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SHIFTING LAKE TURBIDITY

ALTERNATIVE STABLE STATES IN SHALLOW LAKES

Shallow freshwater lakes provide critical habitat for wildlife and recreational opportunities for the public. They're particularly vulnerable to nutrient runoff, which can abruptly shift water conditions from clear to turbid. Turbidity kills fish, degrades habitats and food sources, and negatively impacts recreation and tourism. Worldwide, managers have made considerable efforts to restore turbid shallow lakes to their clear state, but many fail to recover to their original state even when nutrient inputs are controlled. Mathematics has the potential to provide fundamental insights into the mechanisms driving these shifts and support effective, cost-efficient and sustained approaches to restore clarity to these important ecosystems.

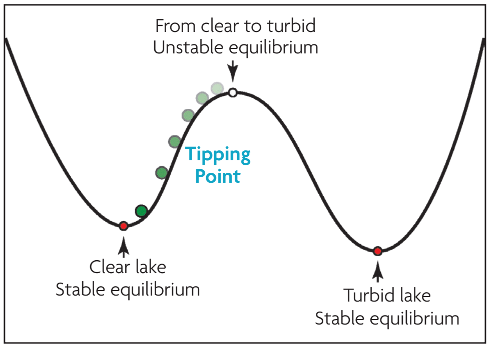
KEY MESSAGES

- ☑ Shallow lakes are characterized by two alternative stable equilibria: a clear state and a turbid state. Perturbations such as an increase in nutrient inputs can cause a lake to shift abruptly from clear to turbid.
- ☑ Lakes will stay in one of the two states, stabilized by feedback loops. Understanding these feedback loops helps to identify leverage points to control the system and generate transitions to the desired stable equilibrium.
- ☑ A small increase in stressors can trigger sudden, major changes in the system that can be difficult to reverse; the system has crossed a tipping point.
- ☑ When conditions change, the clear, stable equilibrium may disappear, thus preventing restoration of the lake, a phenomenon called hysteresis.
- ☑ Mathematics can help support successful and sustained management of these important ecosystems by providing fundamental insights into their dynamics and vulnerabilities.

CLEAR TO TURBID

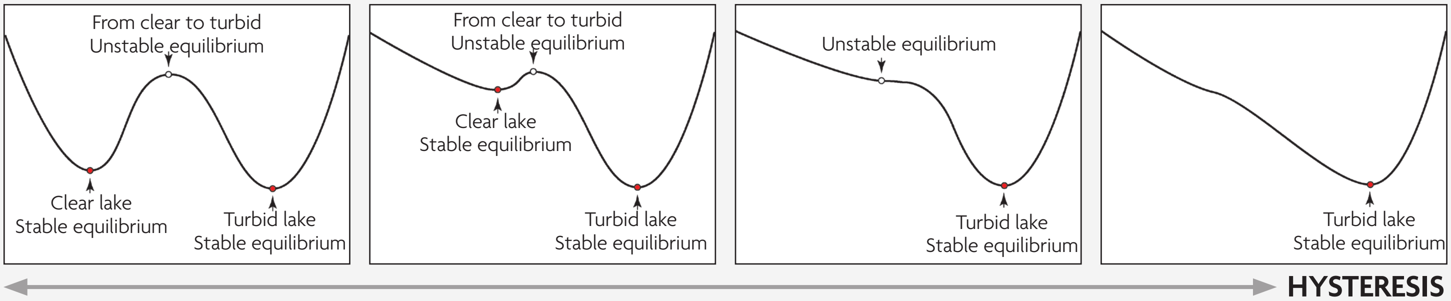
Phosphorous inputs from over-fertilized farmlands fuel the growth of algae and increase turbidity. It stands to reason that if phosphorous inputs and outputs are equal, lake ecosystems will exhibit equilibrium turbidity. In reality, two stable equilibria exist, one clear and one turbid.

Imagine the shallow lake is a ball rolling along a curve shaped like two valleys separated by a hill. The ball will be attracted to one of the two valleys representing the clear and turbid stable equilibria. If the ball is pushed up and over the hill — the tipping point — it will roll downhill into the right-hand valley. Once the ball falls into the bottom of the curve, it tends to stay there, stabilized by mechanisms or feedback loops. It will take more force to push the ball up and out of the valley than to prevent it from falling in the first place.



CATASTROPHIC TRANSITIONS

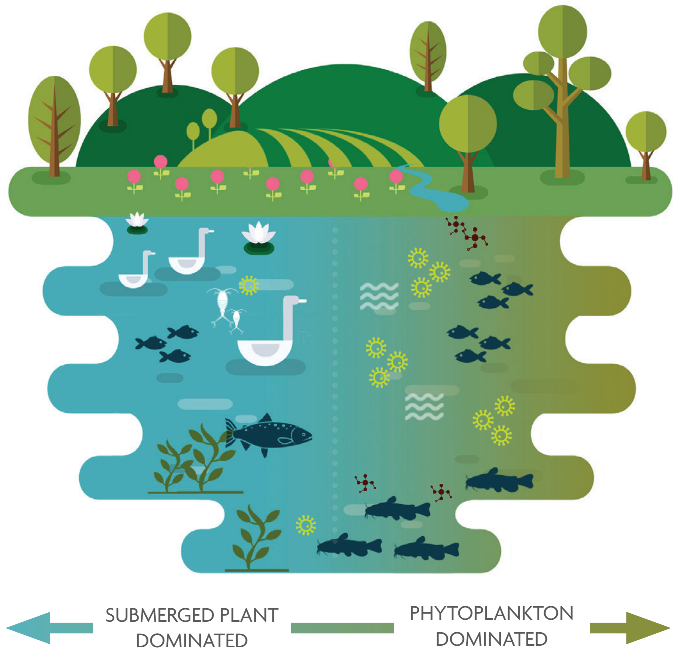
Suppose area farming intensifies, dramatically increasing phosphorous inputs, at the same time water levels fluctuate due to climatic events. These extreme perturbations could shift the profile of the curve. Once on the right, the system exhibits hysteresis. No management action will be effective in restoring the clarity of the lake. Success would require shifting the slope profile back to the left.



FEEDBACK LOOPS

CLEAR LAKE
FEEDBACK LOOPS

- Aquatic plants reduce phytoplankton growth by decreasing phosphorous availability and providing escape cover for zooplankton.
- Adequate zooplankton populations keep phytoplankton populations in check.
- Aquatic plants prevent particle resuspension by stabilizing the lake bed and attenuating wave action.



TURBID LAKE
FEEDBACK LOOPS

- Turbidity blocks underwater light, preventing the development of large, rooted aquatic plants.
- Without the refuge of plants, zooplankton are over-consumed by fish. With fewer zooplankton, phytoplankton grow explosively.
- Unprotected sediment is resuspended by wave action and by bottom-feeding fish, increasing turbidity and releasing legacy phosphorous from sediments.

Given these forces, restoring clarity to a shallow lake is challenging. But it is possible if feedback loops are neutralized. For example, removing bottom-feeding fish prevents sediment resuspension and allows plants to root.

CONCLUSIONS

Identifying tipping points, feedback loops, and their drivers is essential to anticipating and responding to disturbances that might cause a system to fall into an undesirable state. Mathematics can help support the successful and sustained management of ecosystems by providing fundamental insights into their dynamics and vulnerabilities. Examples include:

- ***Identifying reliable early-warning signals*** for approaching tipping points
- ***Predicting success or failure*** of restoration before investments are made
- ***Identifying cost-efficient strategies*** for safeguarding ecosystem services

Tipping points, feedback loops, and hysteresis phenomena have been documented in ecosystems worldwide. Classic examples include coral reef die-offs, desertification, melting of the Arctic ice pack, and fisheries collapse.

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RECKONING WITH UNCERTAINTY

SOCIAL SCIENCES LESSONS FOR MATHEMATICAL MODELING

The COVID-19 pandemic has brought mathematical modeling to the forefront of public attention and debate. Even the simplest of epidemiological models have played an essential role in informing decision-making and society at large. Vocabulary such as 'flattening the curve' has become part of the collective lexicon. But with popularity comes criticism and dissent, primarily when models are used to take unpopular decisions like containment policies. Models are mathematical constructs better understood by their developers than by users. So should the public trust models? The social sciences offer insights that can help society demand the quality it needs from modeling.

KEY MESSAGES

- ☑ Mathematical models can serve society well, as in the example of meteorological forecast models. But not all models are useful. Simple rules can benefit both models and their relationship with society.
- ☑ Model results are conditional on modeling assumptions. The potential outcomes that models project depend on the assumptions they make. Even the best models are affected by uncertainties that aren't always easy to recognize, understand, acknowledge, or communicate. Opacity about uncertainty damages trust.
- ☑ Modelers need to more effectively and transparently communicate the proper uses and limitations of their models to decision-makers and the public. Likewise, modelers need to communicate an appreciation for, and the public needs to accept, what the numbers in those models really mean and do not mean.

Used appropriately, mathematical models serve society exceptionally well. Perhaps the best-known models are those used in weather forecasting, which provide critical data for transportation, travel, disaster prevention, and for simply planning outdoor picnics. Unfortunately, not all models are up to this same standard of societal effectiveness.

When social scientists look at mathematical models, they discover a multiverse, where each scientific discipline adopts its own styles of modeling and quality control. Very little in the way of 'user instructions' is available to those affected by modeling practices. In June 2020, a cross-disciplinary group of natural and social scientists published a manifesto in *Nature* that described what is urgently needed to ease

MIND THE ASSUMPTIONS

Uncertainty quantification and sensitivity analysis are complementary approaches to measuring the robustness of model predictions.

The usefulness of a model depends largely on the accuracy and credibility of its outputs. Yet, because model inputs are rarely precise, output values are always subject to some imprecision and uncertainty. Uncertainty analysis is the process of determining the uncertainty in the model output that is generated from uncertainty in parameter inputs. An essential complement to uncertainty quantification is a sensitivity analysis, which involves assessing how variations in model outputs can be apportioned to different input sources. Performing global uncertainty and sensitivity analyses is fundamentally critical to model quality. Conveying the uncertainty associated with model predictions can be as important to decision-making and policy development as the predictions themselves.

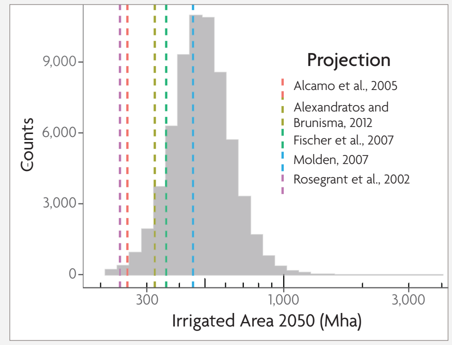
MIND THE HUBRIS

At their core, models are simplified representations of real systems or processes.

It's commonly held that simpler models are often preferable to complex ones. They're easier to understand and validate, and their predictions are typically more accurate. Increasing complexity comes at the cost of adding parameters, whose uncertainty propagates to the model outputs. But this is at odds with current trends that see increasingly complex and larger models. This attraction to complexity may reflect the justified ambition of modelers to achieve a more accurate representation of the study system. But no matter how big or complex the model is, it cannot reflect all of reality. If models are to fulfill their objectives, modelers must resist the urge of complexity as a goal and, instead, build models with an optimum trade-off between complexity and error.

EXAMPLE: MODELING FUTURE IRRIGATED AREAS

In these models, analysts were asked to predict how much irrigated land will be needed by the year 2050. The dashed vertical lines represent predictions from rather complex analyses, without uncertainties attached. The gray histogram represents an uncertainty analysis, where uncertainties from input variables and assumptions are propagated through the model to the output. Most predictions range between 240 and 450 million hectares (Mha), underestimating the potential expansion of irrigation by ignoring basic parametric and model uncertainties. When these are taken into account, the probability distribution of global irrigated land spans almost half an order of magnitude (300–800 Mha), yet higher values, up to 1,800 Mha, cannot be excluded. (*Figure adapted from Puy et al., 2020*).



MIND THE FRAMING

Framing refers to the different lenses, worldviews, or underlying assumptions that guide how individuals, groups, and societies perceive a particular issue.

Model results will at least partly reflect their creators' disciplinary orientations, interests, and biases. Critics of model predictions or policy implications will point to these biases to sow public distrust. How these results are framed and communicated can influence public opinion and steer one policy outcome over another. Modeling practitioners must develop models that are transparent and help model users understand their inner workings and outputs. Successful and transparent framing can support effective results communication and enhance trust with stakeholders.

MIND THE CONSEQUENCES

When appropriately executed, mathematical modeling helps society make smarter decisions. But when not done well, models can lead to wrong or simply unjustified choices.

Quantification can backfire. By helping to make complex financial products seem safe but failing to highlight the underlying assumptions clearly, models contributed to the breakdown of global financial markets in 2008. Society must collectively establish new social norms and ethics of quantification to ensure model predictions contribute to effective decision-making, Modelers must refrain from projecting a false sense of certainty, and decision-makers cannot offload accountability to models just because they fit a pre-established agenda.

MIND THE UNKNOWNS

Failure to acknowledge and communicate uncertainties can artificially limit policy options and open the door to unintended consequences.

Philosophers have long reflected on the virtue of knowing what is not known. German philosopher and mathematician Nicolas Cusa described this in *De Docta Ignorantia* — learned ignorance. Mathematical modeling often sins of excess precision. Too often, modelers are reluctant to acknowledge uncertainties, fearing candor undermines their credibility. In presenting their results, modelers must communicate how prediction uncertainties might change the conclusions. Being transparent about uncertainties strengthens public trust, both in the models and their sources.

the dialogue between models and society. The following five lessons summarize these best practices for responsible mathematical modeling.

CONCLUSIONS

Statistician George EP Box famously said, "Essentially, all models are wrong, but some are useful." Useful models foster understanding. When used appropriately, they make life better and safer in myriad ways. The five lessons above can help ensure mathematical models are responsibly produced and ultimately useful. Each of these lessons showcases the strengths and limits of model outputs and collectively will help preserve mathematical modeling as a valuable tool.

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PREPARING FOR A CRISIS

IMPROVING THE RESILIENCE OF DIGITIZED COMPLEX SYSTEMS

Technology can transform societies, make them more equitable, and support and accelerate the achievement of sustainable development goals. But the rapid and ubiquitous advancement of digital technologies comes with risks. As the COVID-19 pandemic starkly demonstrated, those risks are systemic and often shared in an increasingly connected and globalized world. Governments, institutions, and societies must make investments and systemic changes to reduce the likelihood of future shocks and improve resilience to those shocks. Mathematical methods have proven to be invaluable tools to address vulnerabilities in critical systems and processes and build more resilient systems and societies.

At the start of the COVID-19 pandemic, shortages of personal protective equipment, food items, and other basic consumer goods revealed the risks of over-reliance on 'just-in-time' global supply chains. Companies have been using artificial intelligence and machine learning to minimize raw materials and inventories and only order them when needed to get them to consumers on time. It took a global pandemic to confirm what experts had warned for years: this leaner, lower-cost system was not resilient to shock.

Resilience is a system's ability to minimize or quickly recover from a disruption or shock. Building resilience is the key to combating a crisis, whether environmental, social, or economic. To this end, resilience has been elevated to a global priority, if not a normative imperative, in development discourse and policy agendas.

DIGITAL RESILIENCE

The rapid and global advancement of digital technologies coupled with the availability of vast amounts of data has made societies increasingly dependent on complex systems. From enabling financial transactions to running the power grid, underpinning transportation systems, empowering health care, and supporting the logistics of rapidly delivering supplies and materials, digital technologies have transformed nearly all facets of modern life and society. Yet these changes have made societies more vulnerable to catastrophic disruptions from natural disasters, deliberate attacks, and even simple errors. Making complex systems more resilient is an important challenge as governments, businesses, and society continue to embrace digital technologies.

KEY MESSAGES

- ☑ Resilience is the ability of physical, natural, or social systems to withstand and recover from disruption.
- ☑ Today's highly digitized complex systems have been created through the availability of vast amounts of data, but dependence on such data makes them vulnerable to disruptions due to natural disasters, deliberate attacks, and even simple errors.
- ☑ Mathematics provides capabilities for modeling, simulating, and assessing the behavior of critical infrastructure components and their associated dependencies.
- ☑ Mathematical methods and approaches are invaluable tools to systematically identify threats, develop options that will reduce exposure to or minimize impacts of such threats, and ultimately build more resilient systems. Collectively, these capabilities will help decision-makers prepare and respond more quickly and effectively to a variety of disruptions.

DISRUPTION & RESTORATION

The ability to respond quickly to disruption is key to building resilience. For example, in today's complex, interconnected electrical power systems, cascading failures that result from minor power outages can have dramatic consequences, including widespread blackouts. Power grid disruptions can cascade with such speed that an operator may not be able to absorb the vast amounts of data describing the changing state of the system nor react fast enough to prevent the cascading disaster before it leads to a major system-wide blackout. In such cases, fast, reliable mathematical algorithms and tools are necessary to detect and respond to such problems. These algorithms are autonomous when needed, able to handle multiple alternative solutions, and agile enough to shift direction if no solution is good. Multiple such tools have been developed and applied to increase and enhance infrastructure resilience.

A CASCADE model, for example, is a simple probabilistic model that can uncover and describe some of the essential features of electric power transmission system blackouts caused by cascading failure. It can predict what components might fail and where load distributions might cause cascading overloads and failures following a disturbance. CASCADE models can be paired with algorithms to analyze the effect of operator actions to minimize the impacts of blackouts, such as emergency load shedding, and evaluate expected blackout costs.

Mathematical models have also been applied to recovering critical infrastructure following large-scale disruptive events. Increasingly digitized transportation systems, telecom networks, water and sewer systems, and electric power grids supply crucial services to communities and ensure human health and well-being. These services must be efficiently and effectively restored following a natural disaster or other extreme event. Network optimization techniques can be used to model the resilience of these critical infrastructure systems.

After a disruption, decision-makers must schedule repairs by allocating scarce resources. They need to determine the set of components that will be temporarily installed or repaired, assign these tasks to workgroups, and then determine the schedule of each workgroup to complete its assigned tasks. These planning and scheduling decisions can be mathematically modeled and optimized within a scheduling framework.

After an extreme event, there are often interdependencies between restoration efforts and critical infrastructure systems. For example, repairs may be needed at transportation hubs after a hurricane, but these jobs cannot be completed until power is restored. Or, if trees bring down power lines onto a road, the power company must wait for road crews to begin repair and restoration. Complex mathematical theories to deal with interdependencies have been developed using optimization methods. Often these interdependencies cross different infrastructures, requiring decentralized decision-making and scheduling.

RESILIENT SOLUTIONS

Digital resilience requires design strategies that safeguard a system's ability to maintain, change, and recover capabilities and withstand crises and shocks. Resilience of digitized complex systems must be achieved as a designed-in property. It's possible to mathematically represent the system's structure and operational logic and quantify readiness to adapt and recover. Mathematical and algorithmic approaches can also inform crisis response and recovery efforts and help advance more resilient solutions.

CONCLUSIONS

Modern digital systems tend to include more and more interconnected components. They enhance the ability to accommodate rapid changes in transportation systems, urban services, and supply chains. Designing ways to create flexibility while minimizing vulnerability to disruptions

Artificial intelligence is being used to increase reliability and reduce losses and accidents during the transmission of electrical energy. © AdobeStock

requires a deeper understanding of the dynamics of interconnected complex systems.

Monitoring highly specialized digital infrastructure systems involves the collection of vast amounts of data from multiple heterogeneous sources in real-time. The avalanche of data requires increasingly abstract mathematical models and matching algorithmic techniques to extract information from data that is relevant for decision-makers.

While most intuitively understand what resilience is, decision-makers need to go beyond intuition; they need tools to measure resilience and to assess the effect of specific actions on the resilience of a complex digitized system. Active collaboration of mathematical scientists and decision-makers can help develop such tools.

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VALUING NATURAL CAPITAL

QUANTIFYING THE VALUE OF ECOSYSTEM SERVICES

Human health and livelihoods depend on Earth's ecosystems and the goods and services they provide, from food and water to climate regulation, crop pollination, and aesthetic enjoyment. Yet, despite these vital contributions, more than 60 percent of the world's ecosystem services are being degraded or used unsustainably, according to the Millennium Ecosystem Assessment. Mathematical approaches can help strengthen assessments, project future changes in ecosystem services, and provide policy-relevant information to ensure benefits to future generations.

Nature provides humans with goods and services like clean water and air, food, storm and flood protection, climate regulation, raw materials, energy, recreation, and much more. Despite the enormous benefits of these services to human health, wealth, and well-being, natural capital is systematically being degraded worldwide. The Dasgupta Review, a study published by the U.K. government in 2021, found that global natural capital per capita has declined by almost 40% between 1992 and 2014, even as produced capital per person doubled.

Historically, ecosystem services have not been valued in conventional economic analyses, leading to a misconception of the fundamental role of nature in economies. Quantifying the value of natural capital and the ecosystem services it provides ensures this value is integrated into policy and development planning.

MATHEMATICAL TECHNIQUES

Mathematicians study ecosystems using models and data. The simplest conceptual models focus on the mechanisms driving particular phenomena. At the other extreme are complex process models containing a system of mathematical equations with several known and unknown parameters — usually solved on a computer — to make quantitative predictions about some aspect of a real process.

Mathematical techniques used in the development of these models come from multiple areas of mathematics, including optimization theory, the theory of differential equations, and the theory of dynamical systems.

Once a model has been validated and calibrated with available benchmark data, it becomes a tool to explore what-if scenarios. By illuminating critical features of a particular phenomenon, models improve societal understanding of complex systems and enable better decisions and policies.

KEY MESSAGES

- ☑ Ecosystem services are essential to civilization. Natural capital and the ecosystem services they provide are the very foundation of all human and economic activities.
- ☑ Human activities are impairing the flow of ecosystem services on a large scale. Decision-makers require tools and data to credibly measure the value of vital ecosystem services.
- ☑ Conceptual models at appropriate spatial and temporal scales provide insight into the mechanisms driving the flow of ecosystem services.
- ☑ Mathematical models enable the exploration of multiple "what-if" scenarios. The outcomes of such explorations can illuminate salient features of the phenomena of interest and inform the decision-making process.

STATIC VS. DYNAMIC

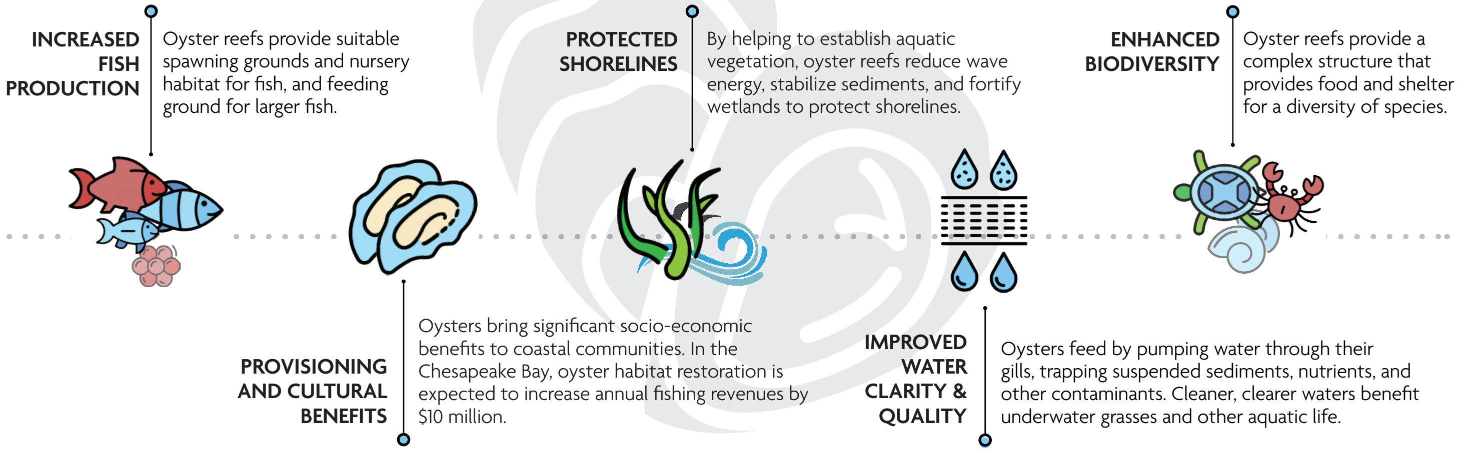
Many early natural resource models were formulated as static optimization problems, which failed to account for the fact that natural capital, like other capital stocks, must be optimally managed over time to reach its full potential value to society. To value natural capital and ecosystem services requires solving a dynamic optimization problem. Economist Jon Conrad illustrated this by estimating the value of oyster-supplied nutrient removal in the Chesapeake Bay. In this simple dynamic optimization model, the costs of an open-access oyster fishery in terms of foregone net revenue and loss of water quality services would range between USD 37 million and USD 79 million per year. Though simplistic, the model demonstrates that steady-state optimums can serve as a benchmark from which to estimate the costs of resource degradation and the benefits of conservation and restoration.

MEASURES OF ECONOMIC PERFORMANCE

Standard measures of per capita Gross Domestic Product (GDP) and the UN Human Development Index (HDI) are

EXAMPLE: THE ECOSYSTEM SERVICES SUPPLIED BY THE CHESAPEAKE BAY'S OYSTER REEFS

The Chesapeake Bay is the largest estuary in the United States and important for both the ecology and economy of the Mid-Atlantic coastal states. The eastern oyster is one of the Bay's most iconic species. In addition to the direct economic benefits of the fishery, oysters provide a multitude of ecosystem services, from providing shelter for blue crab and fish to protecting shorelines. Chesapeake Bay stakeholders and decision-makers use a suite of mathematical and bio-economic models to guide decision-making and to assess current management efforts to protect critical habitats and ecosystem services. These models are among the most sophisticated, studied, and respected in the world.



commonly used to measure a country's level of economic activity and progress in development. Many analysts have argued that these conventional measures can give a highly misleading impression of economic and human development.

Neither GDP nor HDI accounts for environmental degradation and natural resource depletion. If a country clear cuts its forests, pollutes its waters, or depletes its fisheries, it is made poorer, despite any positive contributions of these marketable resources to GDP.

Leading economist Sir Partha Dasgupta has long argued that traditional economic measures must be adjusted to reflect the importance of natural capital. In an effort to move beyond GDP in tracking global progress, the United Nations launched the System of Environmental and Economic Accounts (SEEA), a new international standard for natural capital accounting. The framework integrates economic and environmental data to provide a more comprehensive and multipurpose view of the interrelationships between the economy and the environment. The ecosystem accounts produced by countries will track the extent, condition, and services provided by ecosystems in the form of physical and monetary accounts and indicators. Mathematical and statistical approaches, ranging from more straightforward structural analyses to complex modeling, underpin SEEA applications and extensions.

In April 2021, the United Nations and the Basque Centre for Climate Change launched an innovative artificial intelligence tool to make it easier for countries to measure nature's contributions to their economic prosperity and well-being. The new tool lowers the barriers to compiling SEEA ecosystem accounts and enables a rapid and standardized yet customizable ecosystem accounting for any terrestrial area in the world.

CONCLUSIONS

Over the past 50 years, humans have rapidly and extensively changed ecosystems, largely to meet growing demands for food, fiber, and fuel. The full costs of the loss and degradation of ecosystem services are difficult to measure but likely substantial and growing. To reverse the loss and degradation of ecosystem services, policymakers' economic development motivations must include a conservation objective and ecosystem services values need to be incorporated into any decision-making. Mathematical approaches will be needed to design and strengthen tools to measure and quantify ecosystem services, explore 'what-if' scenarios, and provide knowledge and information to enhance decision-making in support of sustainable development goals.

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ALLOCATING SCARCE RESOURCES

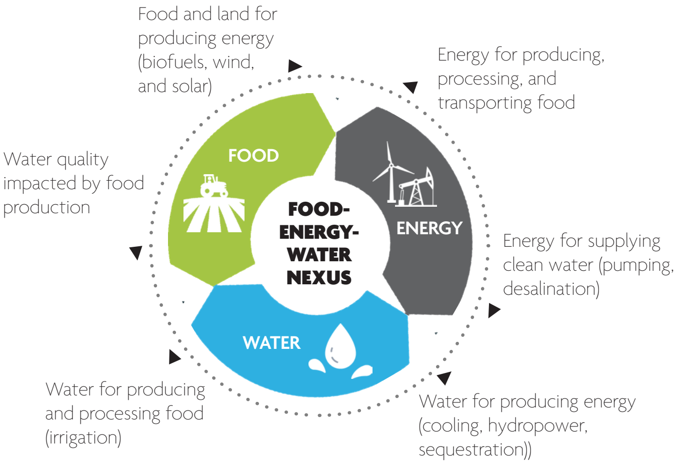
MODELING TO SUPPORT FOOD-ENERGY-WATER SUSTAINABILITY

Global freshwater supplies are increasingly under stress due to the combined effects of population growth, increasing industrialization, and climate change. As a result, water resources must be managed and allocated in ways that are economically efficient and that account for interdependencies between food production, energy generation, and water provision, often referred to as the food-energy-water nexus. Mathematical modeling provides the means to anticipate what might happen in the future based on certain conditions, allowing decision-makers to optimize water use across sectors while taking into account uncertainties over future water availability.

KEY MESSAGES

- ☑ An integrated approach to food-energy-water nexus management ensures solutions that are more favorable to sustainability than those made in silos. In the face of huge increases in demand for food, energy, and water over the next decades, nexus management will need to be more efficient across sectors and regions to enable optimal use of limited resources.
- ☑ Future water availability is highly uncertain as it depends on weather variability and climate change. Ignoring uncertainty can increase both costs and risks. Uncertainty should be taken into account in decisions to invest in new technologies and solutions with a longer time horizon for return.
- ☑ Mathematical modeling offers options for optimizing upfront investments in water-saving technologies and water use to ensure the desired level of domestic food and energy security given the uncertainties.

Food, energy, and water are central to sustainable development, yet food and energy compete for water in many parts of the world. Agriculture is the largest consumer of the world's freshwater resources, and the food sector currently accounts for roughly 30 percent of global energy consumption. Too often, these domains are managed in silos, undermining security in the other domains, and jeopardizing human well-being and economic growth. The intertwined nature of food production, energy generation, and water networks — known as the Food-Energy-Water Nexus — calls for an integrated approach to management and decision-making. By holistically managing this nexus, decision-makers can better manage trade-offs and mitigate uncertainty and risks.



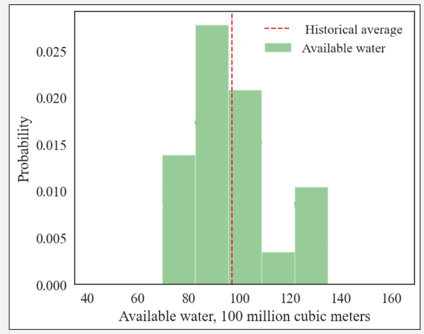
To advance sustainable management of food-energy-water nexus systems, scientists develop computer models that analyze alternative system configurations, such as combinations of technologies that can be deployed in energy and agriculture, types of crops that can be grown, and the use of fertilizers. Typically a model implements a particular management goal, for example, minimizing costs or maximizing profits. A model's solution is a system configuration that achieves the desired goal given food, energy, and water demand targets and supply constraints.

CHALLENGES OF UNCERTAINTY

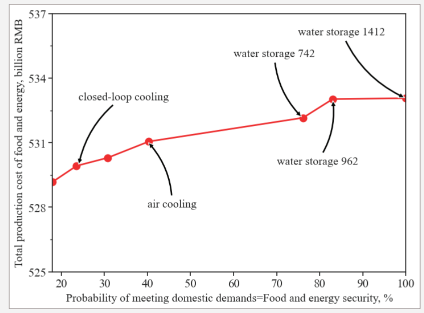
Food-energy-water systems are affected in significant but uncertain ways by a number of complex external drivers, including financial markets, human preferences and behavior, and, increasingly, weather variability and climate change. Both precipitation and evaporation influence patterns of water availability, and climate change is making corresponding weather conditions more severe and less predictable. Crop choices and other crop management decisions are particularly challenging when little is known about future drought risk or water demand. This challenge is especially evident in developing countries whose economies are often highly dependent upon agricultural production and export.

CASE STUDY: SHANXI, CHINA

Water scarcity is a pervasive problem across much of China. Population growth, rising food demand, and disrupted rain and snowfall patterns due to climate change further compound the problem. The coal industry is responsible for more than 22 percent of the nation's total water withdrawal, second only to irrigated agriculture at over 60 percent. Shanxi, China's top coal-producing province, is one of the most water-scarce provinces in the northern part of the country. Decades of mining have damaged underground water tables and contaminated groundwater supplies. As a result, residents are forced to draw on groundwater, which is often pumped faster than it can be recharged. Dry years can cause reservoirs and rivers to dry up and groundwater levels to subside, further limiting drinking water supplies.



A team of scientists used stochastic, chance-constrained programming models to ensure food and energy security in the region through water-use efficiency in coal production. Higher levels of food and energy security require greater investments in water-saving technologies that allow water to be used more efficiently and therefore ensure that domestic production can meet demand even when the water supplies are low. As finances are limited, the model offered solutions that prioritize introducing water-saving technologies over time, starting with those that deliver higher-security dividends at a lower cost. Both closed-loop cooling and air cooling technologies reduce water use in coal-fired power plants compared to the widely-used open-loop cooling. Transitioning to these water-saving technologies could free up water for irrigated agriculture. Deploying closed-loop cooling in all of Shanxi's coal-fired power plants would support meeting domestic food and energy demands with a probability of 24%. The deployment of air cooling would increase this probability up to 40%.



Traditional food-energy-water decision-support models rely on average parameter values derived from historical data. However, this approach is not without risk. For example, in years with below-average water, solutions may require more water than is available. Decision-makers would have to either reduce production targets and run the risk of food and energy under-supply, prioritize production and take water away from private households, or exploit the water system beyond sustainable levels. Any of these measures alone or in combination would ultimately reduce human well-being.

OPTIMIZING UNDER UNCERTAINTY

Stochastic programming is an increasingly popular mathematical tool for modeling problems that involve uncertainty. When model parameters are unknown or uncertain, modelers turn to probability distributions — a function that shows the relationship between the outcome of a plausible event and its frequency of occurrence. To maximize a set of benefits while minimizing risks, these models can include 'chance' constraints, which ensure that the probability of meeting a specific constraint is above a certain level. At the food-energy-water nexus, chance-constrained optimization would allow decision-makers to set up food and energy security goals based on the target probabilities for which the domestic supplies of food and energy would meet the domestic demand. These approaches have been especially important in engineering and finance, where uncertainties in price, demand, supply, and currency exchange rate are common. More recently, it's been used to manage water resources and agricultural supply chains and optimize renewable energy portfolio requirements.

CONCLUSIONS

Chance constrained stochastic programming is an effective and convenient approach to control risk in decision-making under uncertainty. It improves the realism of model recommendations as it avoids the fallacy of relying on averages. Sustainability transitions rely on processes that involve numerous uncertainties, and these must be addressed in models used to inform such transitions. Unique mathematical approaches allow the inclusion of probabilities to meet desired objectives, giving decision-makers additional space to maneuver in reaching sustainability goals.

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MODELING CLIMATE

THE ENERGETICS OF A WARMING WORLD

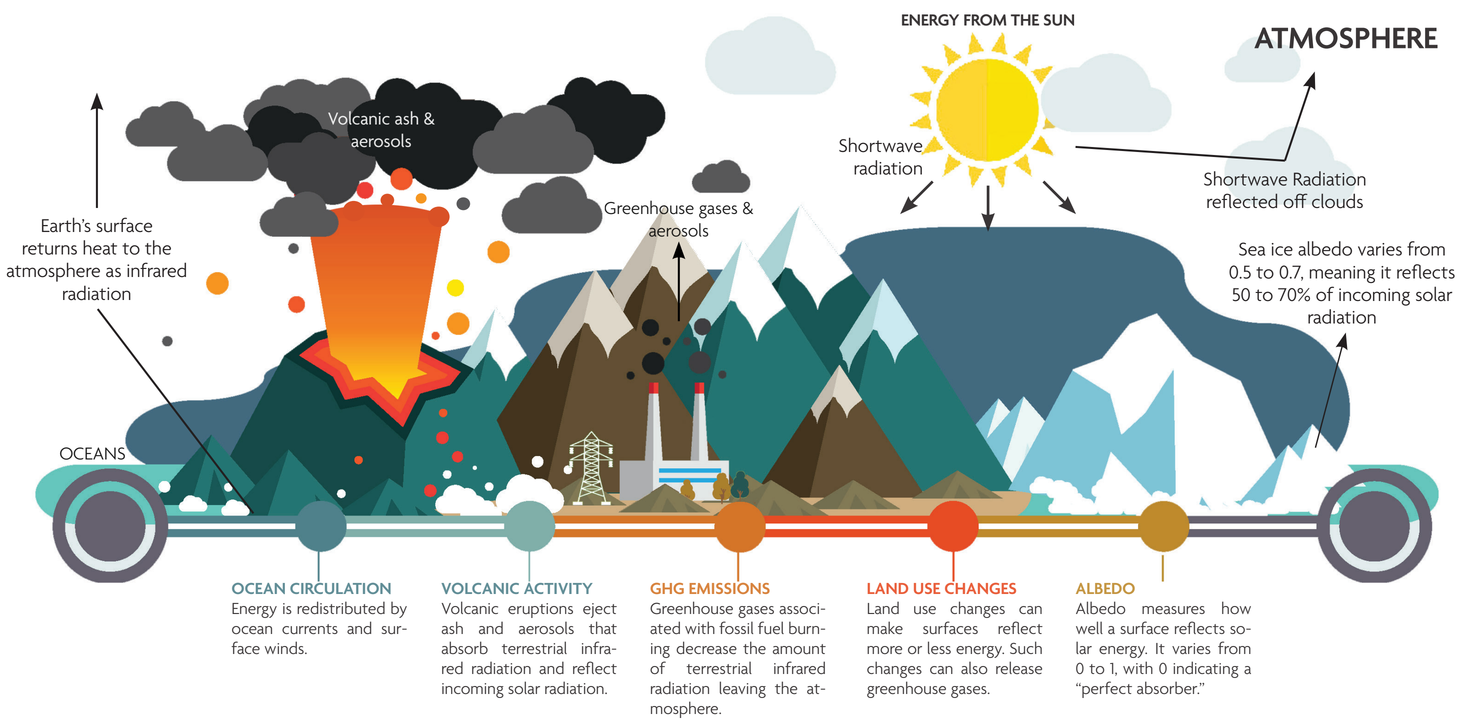
A simple but powerful mathematical model can provide insights into the past, present, and future of Earth's climate. The energy-balance model estimates the Earth's surface temperature based on changes in the amount of energy coming in and going out of the climate system. Such models allow climate scientists to test how the Earth's surface temperature responds to hypothetical changes, both natural and human-caused. Results from climate models help explain past and future climate systems and inform decisions about mitigation and adaptation strategies.

KEY MESSAGES

- ☑ The Earth's atmosphere acts like a massive heat engine, fueled entirely by energy from the sun. To maintain a constant global average temperature, all of the sun's energy that enters the atmosphere must eventually be sent back out.
- ☑ Climate models use mathematical expressions to simulate how energy and matter interact in Earth's subsystems. They range from simple, stripped-down conceptual models to state-of-the-art Earth system models.
- ☑ A simple energy-balance model can be used to test the response of Earth's surface temperatures to external changes, both natural and man-made.
- ☑ Results from climate models provide robust information to help governments, policymakers, and the public understand how Earth's climate changed in the past, how it is changing now, and what the future might hold.

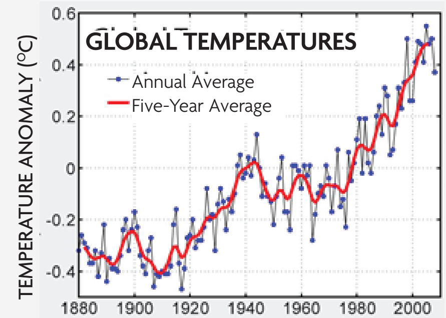
EARTH'S CLIMATE SYSTEM

The Earth's climate system comprises five major components: The atmosphere, oceans, ice and permafrost, land surfaces, and all living organisms, including humans. The atmosphere acts like a massive heat engine, fueled entirely by energy from the sun. Maintaining a constant global average temperature requires that all of the sun's energy entering the atmosphere must eventually be sent back out. If too much energy is reflected back to space, the system cools; it warms if if too much energy is absorbed. Because the Earth's surface is heated unequally – with more sunlight hitting the equator than the poles – the planet balances out these energy differences by constantly redistributing heat via surface winds and ocean currents.



CHANGING CLIMATE

Earth's climate is never static; it changes as the system's various components change and interact. While climate has changed dramatically many times over the planet's 4.5-billion year history, most changes to the global mean temperature occurred slowly over tens of thousands or even millions of years. Most of these changes are attributed to small variations in Earth's orbit that affect the amount of the sun's energy entering the atmosphere. However, over the past century, global average temperatures have increased at an unprecedented rate, driven largely by human activity.



Source: NASA's Goddard Institute for Space Studies

Various external drivers can change the Earth's energy balance. These drivers, known as 'climate forcings,' can be natural and human-caused. For example, natural volcanic eruptions alter the Earth's energy balance by ejecting aerosol clouds of gas and ash that absorb terrestrial radiation and scatter incoming solar radiation. Human-caused or anthropogenic forcings include emissions of heat-trapping greenhouse gases and land-use changes that make land reflect more or less solar energy. Human-caused forcings have been increasing since 1750, and their effect now dominates all natural climate forcings.

MODELING COMPLEX SYSTEMS

Complex systems — like Earth's climate system — cannot simply be deconstructed into a set of individual parts but must be viewed holistically as a system of interactions between and within component objects. As such, they are challenging to model.

Climate models use mathematical expressions to simulate how energy and matter interact in Earth's subsystems. They range in complexity from simple, stripped-down conceptual models to state-of-the-art Earth system models requiring supercomputers to run. Scientists use the results to project future climate conditions under various scenarios, providing governments and policymakers with robust information to evaluate and implement mitigation and adaptation strategies.

THE ENERGY BALANCE MODEL

The energy balance model is the simplest possible description of the climate system. It's used to estimate the Earth's average surface temperature by accounting for all energy coming into and out of the system.

The Earth is heated by energy from the sun, and heat energy that is not absorbed is radiated back into space. When the incoming and outgoing energy balance, the system is in equilibrium. Climate scientists can derive the average surface temperature by balancing mathematical expressions for the incoming and outgoing energy.

The model accounts for the impact of albedo, a measure of how well a surface reflects solar energy. Snow and sea ice, for example, have a high albedo as most of the sun's energy is reflected off their surfaces without being absorbed. More sophisticated energy-balance models also account for the greenhouse effect — whereby the Earth's surface has to warm up so that it will emit more infrared energy to compensate for the effect of added greenhouse gases.

Energy-balance models allow scientists to test how the Earth's surface temperature responds to hypothetical changes in natural and human-caused forcings. For example, if sea ice melts, albedo decreases, which would increase temperature. If carbon dioxide emissions increase, atmospheric emissivity — the transparency of the atmosphere — would decrease, again forcing temperature higher. With only a limited number of variables and parameters, this simple conceptual model can provide powerful insights into future climate.

INCREASING COMPLEXITY

To express climate in a region or continent, scientists can input these energy-balance equations into box models. Each box represents a column of the climate system extending above a specified surface area. They can also include and combine certain land, ocean, and ice parameters to simulate large-scale climate scenarios. General circulation models are the most complex and precise and incorporate a wide range of physical processes, including ocean circulation and atmospheric chemistry, represented through millions of variables and parameters.

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FACING FUTURE CLIMATES

CLIMATE MODELING FOR IMPROVED DECISION-MAKING

Human-induced climate change has caused rapid, dramatic, and unprecedented changes to the planet. No country is immune to the human health and societal impacts of climate change, but the world's poorest countries have only limited capacity to adapt to these impacts and respond to the damages. Mathematical tools, methods, and theories are helping scientists understand how natural and man-made influences affect Earth's climate. With increasing detail and precision, model outputs provide information for policy-relevant and regionally-specific decision-making. Such information will enable countries to scale up and accelerate adaptation and disaster risk reduction activities.

The Intergovernmental Panel on Climate Change's 2021 report, authored by hundreds of the world's top scientists and signed off by all 195 member countries, has concluded that climate change is occurring and it's unequivocally caused by human activities. Fossil fuel burning, deforestation, and other human activities release greenhouse gases that trap the sun's heat, warming the atmosphere and the Earth's surface below. Since the 19th century, humans have already heated the planet by roughly 1.1 degrees Celsius. It's already affecting every region on Earth and the whole of the Earth's climate system. Many of these changes are unprecedented in the last 2,000 or more years.

Mathematical models that simulate Earth's climate are central to climate research. For the past 50 years, they have allowed scientists to better understand the climate system, test hypotheses and draw conclusions on past and future climate systems, and prioritize appropriate human responses.

KEY MESSAGES

- ☑ Mathematical tools and theories are fundamental to understanding climate change and anticipating its risks. They're foundational to predicting impacts, informing mitigation and adaptation decisions, and setting climate policy targets on national and international scales.
- ☑ State-of-the-art mathematical models that simulate the Earth's climate system enable scientists to predict, from fundamental physics laws, how the climate will evolve in the future.
- ☑ Climate models are fed by Shared Socio-Economic Pathways scenarios of radiative forcing based on ways society might change over the next century.
- ☑ The Climatic Impact-Driver (CID) framework helps bridge model outputs and climate actions. CID-relevant information informs climate services, the assessment of climate-related risks, adaptation planning, and decision-making.

CLIMATE MODELING

Climate models are systems of differential equations based on the basic laws of physics, fluid motion, and chemistry. They characterize how energy and matter interact to drive the different components of the Earth's climate system: the atmosphere, oceans, biosphere, and land- and ice-covered regions of the planet.

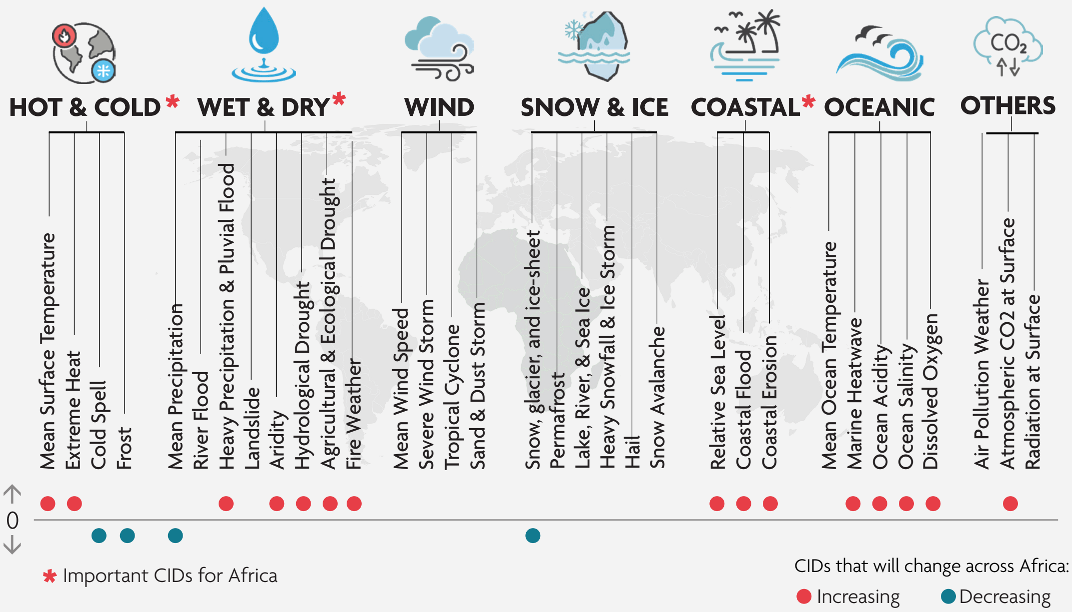
Rather than treating the Earth as a whole, climate models use numerical methods that divide it into a series of small, three-dimensional grid cells. The size of these grid cells defines the model's spatial resolution. Higher-resolution models have more and smaller boxes. The model also divides time into smaller time steps. A smaller time step provides more detailed climate information but requires more calculations with every run. The climate model calculates the state of the climate system in each cell and evaluates interactions with neighboring grid cells. By numerically solving the equations that capture the climate's underlying mechanisms, climate models simulate how conditions in each cell change over time.

The main inputs into climate models are the forcing agents that change the Earth's energy balance and contribute to climate change. They include solar radiation, greenhouse gases, aerosols, volcanic activity, and land-use change. These exert a radiative forcing or heating effect that will change temperature, winds, precipitation, and other parameters in the climate system. These climate forcings are run through models to estimate past conditions or develop future climate change scenarios.

FUTURE SCENARIOS

There is uncertainty in future population and economic growth, education, urbanization, and the rate of technological development. These uncertainties may result

CASE STUDY: CLIMATIC IMPACT-DRIVERS IN AFRICA



Climate models project that by mid-century, multiple climatic impact-drivers will change across Africa. In all regions, mean temperatures and extreme heats will increase, and cold spells will decrease. The frequency and intensity of heavy precipitation events are projected to increase in nearly every region. Changes to the magnitude, frequency, duration, seasonality, and spatial extent of these CIDs will threaten human health, food and water security, and socio-economic development. Paired with climate services, the CID framework provides useful information for climate-resilient development planning and practice.

in very different climate forcings. To explore and evaluate possible future climates, scientists use climate models in combination with storylines to produce scenarios of plausible alternative futures.

In 2014, researchers released the Shared Socio-Economic Pathways (SSPs), a new set of alternative pathways of future societal development. The five SSPs describe a range of plausible trends in radiative forcings over the next century. Mathematical models use these scenarios to explain how the Earth's temperature and precipitation will change under the different SSP pathways. The stronger forcing scenario results in a larger temperature change and vice-versa.

CLIMATIC IMPACT-DRIVERS

In its sixth assessment report, the Intergovernmental Panel on Climate Change introduced the Climatic Impact-Driver (CID) framework to help translate model outputs of physical climate conditions — averages, events, and extremes — into what they mean for society and ecosystems. The 33 CIDs and their changes can be detrimental, beneficial, neutral, or a mixture of each. The CID framework provides decision-makers with actionable and risk-relevant climate information to inform regional adaptation, mitigation, and risk planning.

CLIMATE SERVICES

Climate services provide CID-relevant information in a way that assists civil society and government organizations in making improved adaptation and risk management decisions. Depending on the user's needs, this data and information may be combined with non-meteorological data, such as population, agricultural production and health trends, and other socio-economic data. Such services are most effective when developed in collaboration with users and ensure users can access, interpret, and utilize the information.

CONCLUSIONS

Mathematical methods, tools, and theories play a vital role in climate research. To date, mathematical modeling remains the only effective approach for predicting the evolution of the Earth's climate system in response to both natural and man-made influences. Despite the enormous advances in the past 50 years, more sophisticated mathematical approaches are needed to revolutionize the quality of information available for mitigation and adaptation decision-making.

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FORECASTING CYCLONES

THE MATHEMATICS OF TROPICAL CYCLONE PREDICTION

Tropical cyclones are among the most damaging and destructive weather events, second only to earthquakes in terms of worldwide fatalities. Early and accurate forecasting allows emergency management officials, the private sector, and the general public to make more informed decisions during major storms, minimizing the losses of life and property. Improvements in mathematical prediction models, coupled with real-time data collection and analysis, have dramatically improved tropical cyclone track forecasting, such that today's average 5-day forecast is as accurate as a 3-day forecast was ten years ago. Continued efforts to improve track forecasts may also prove useful for predicting cyclone intensity.

KEY MESSAGES

- ☑ Tropical cyclones are among the biggest threats to life and property across the world. In addition to high winds and storm surge, they threaten coastal and inland communities with torrential rains and flooding. Improved track and intensity predictions provide invaluable early warnings for vulnerable communities and data-informed support for emergency and evacuation decision-making.
- ☑ Improvements in tropical cyclone track forecasting have been significant, arising from improved observations and models, and better integration of these through data assimilation.
- ☑ Accurate forecasts of tropical cyclone intensity remain challenging. A mathematical understanding of the physical mechanisms of tropical cyclone dynamics may prove valuable to understanding how and why cyclones intensify and improving real-time intensity predictions.

Tropical cyclones originate over warm ocean waters and develop into huge, organized cloud systems circulating around a low-pressure center. Their intense and often devastating rotation results from the Coriolis Effect, which deflects the storm's winds to rotate counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere.

As they draw energy from the ocean, particularly from warm water, tropical cyclones occur primarily between the Tropics of Cancer and Capricorn. Intense tropical cyclones occurring in the Atlantic Ocean, known as hurricanes, can affect the Caribbean and the United States. In the western Pacific, they're known as typhoons and can impact the coastal regions of Mexico, southeast Asia, Australia, and the South Pacific islands. Cyclones that form in the Indian Ocean can strike India, Bangladesh, Tanzania, Mozambique, Mauritius, and Madagascar.

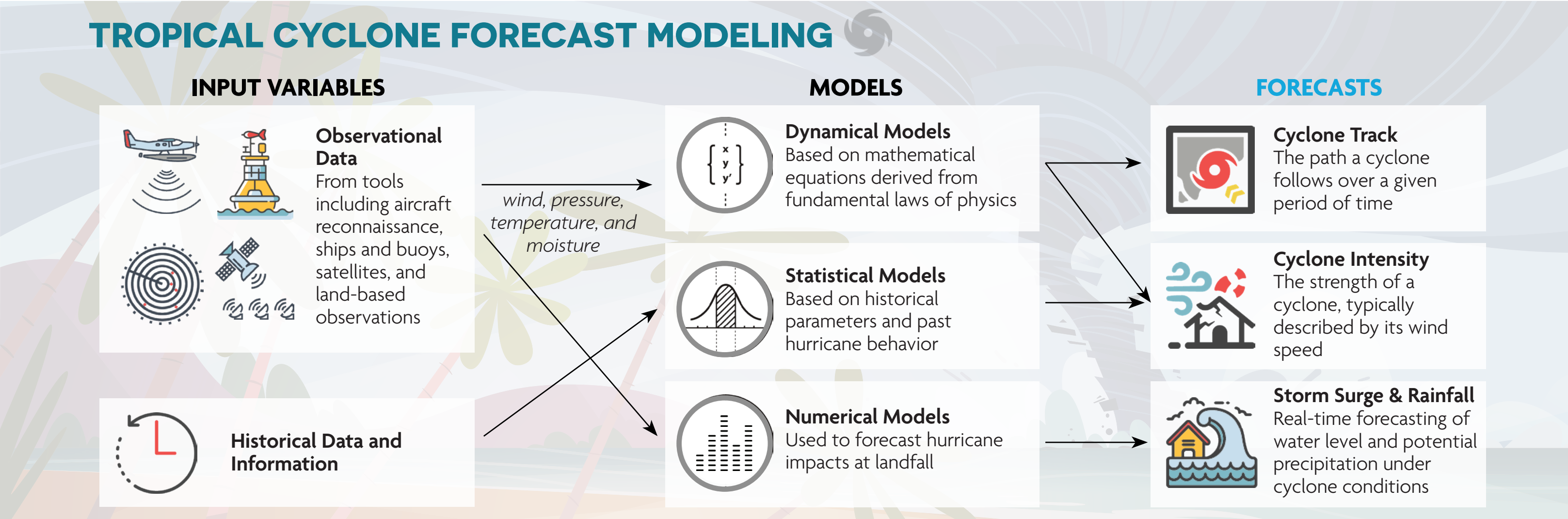
Over the last 20 years, powerful winds and heavy rainfall produced by cyclones have killed more than a quarter of a million people, injured or displaced countless others, and they have an annual impact of billions of dollars on the global economy. These impacts are expected to intensify with sea-level rise, warming temperatures, and other climate-mediated changes. This potential for widespread devastation underscores the value of timely and accurate tropical cyclone forecasts for providing early warnings to vulnerable coastal communities.

BEYOND A CRYSTAL BALL

Today, forecast models utilize dynamical models of the atmosphere to predict the future behavior of a tropical cyclone's path and intensity. Foundational to these models are mathematical equations derived from fundamental laws of physics, including laws of motion and thermodynamics. For example, Newton's second law of motion states a force (F) acts on a mass (m) and produces an acceleration (a). This relationship — $F = ma$ — can help predict a cyclone's future wind speed. These laws are expressed in terms of a system of partial differential equations.

Forecasters use observational data from satellites, reconnaissance aircraft, ships, buoys, and radar to describe the initial conditions of the atmosphere and then solve the model's mathematical equations to predict the evolution in time and space of atmospheric conditions, including the density, pressure, potential temperature, and wind velocity and positions.

For predictions about the frequency and intensity of cyclones that will occur in the upcoming season — known as seasonal outlooks — forecasters mainly use statistical models based on historical relationships between meteorological parameters and the behavior of past hurricanes. Statistical models are much simpler and faster to run than dynamical models, but they're generally less accurate.



ADVANCES IN FORECASTING

Tropical cyclone-tracking models have improved steadily in the last half-century. The 2019 five-day track forecast was better than the 1970s 36-hour forecast. Forecast accuracy over an extended time frame has also improved. Today, a tropical cyclone's track can be predicted up to a week in advance, giving cities needed time to evacuate safely. Much of this progress is due to improved mathematical models running on more powerful supercomputers, the availability of real-time, high-resolution data from sophisticated satellites and weather reconnaissance aircraft, and revolutionary data assimilation methods for continuously adjusting models with new observational data.

Predicting the intensity of a cyclone has progressed at a much slower pace. To further improve the accuracy of intensity forecasts, scientists have identified the need for increased model resolution, more powerful computers, enhanced observations, and a better understanding of the cyclone's inner core region.

THE EYE OF THE STORM

A tropical cyclone's inner core, or eyewall, is the area of strongest surface winds and heaviest rains. It surrounds the cyclone's relatively calm eye. A mathematical understanding of tropical cyclone structure and circulation during its lifecycle has led to significant improvements in the design of computer simulation models and forecasts. For example, in the 1950s, Norwegian meteorologist Arnt Eliassen — a pioneer in numerical weather forecasting — developed a classical mathematical model of a tropical cyclone that consisted of a balanced circular vortex embedded in an ambient fluid at rest. Recent mathematical developments have shown that such a vortex represents a natural minimum energy state of the system and can thus explain the damaging inner core often observed in tropical cyclones. Such models also help explain the observed decrease in wind velocity as radius increases outside this core.

CONCLUSIONS

Tropical cyclone forecasting has the potential to save lives and reduce property damage. With a warmer climate, cyclones are predicted to increase in strength. The increase in cyclone intensity combined with growing human populations and other atmospheric changes make understanding and more accurately predicting tropical cyclones a vital area of research. Multiple studies suggest the economic value of improving forecasts has been worth billions of dollars in the last few decades.

There is now an opportunity to engage a wider community of researchers and forecasters to better understand how environmental conditions drive and affect hurricanes and develop and share new forecast approaches and datasets. Collaborations are especially critical for the many smaller and developing countries with limited resources to access or develop reliable, real-time forecasts.

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ATTRIBUTING EXTREME WEATHER

THE NEW SCIENCE OF EXTREME EVENT ATTRIBUTION

Since 1970, extreme weather hazards have occurred every day, on average, over the past 50 years, according to the World Meteorological Organization. Being able to link such hazards to climate change and do so quickly can be an incredibly powerful tool to communicate the urgency and consequences of global warming. The emerging science of extreme event attribution utilizes mathematical approaches to tease out whether and by how much human-caused climate change contributed to individual extreme events. Attribution science has matured to the point where the number and intensity of extreme weather hazards caused by climate change can now be estimated.

CAN I BLAME CLIMATE CHANGE?

The number of extreme weather events has increased fivefold over the last 50 years, driven primarily by climate change. It's only normal for someone affected by a disaster to ask if it was a natural event or a man-made event because of global warming. While trends in vulnerability and exposure are major drivers of disaster impacts, the relatively new technique known as detection and attribution provides quantitative information on the probability and magnitudes of extreme events. Much like an epidemiologist tries to identify the causal factors that contribute to the development or prevention of disease, attribution scientists use mathematical models to test whether — and by how much — human-caused global warming played a part in an extreme weather event.

The science of extreme weather event attribution began after a 2003 heat wave killed as many as 70,000 people across Europe. It was an event hotter than anything recorded on the continent in 500 years. Until that point, detection and attribution studies were limited to quantifying the long-term changes in climate variables, most notably temperature or extreme precipitation. Climate scientists shied away from probing the human effects on individual events. When asked, they'd respond with statements like, "We can't say about this particular event, but such changes are what we would expect." Today, quantitative attribution statements are routinely made for an ever-increasing variety of individual extreme weather events.

KEY MESSAGES

- ☑ The number of extreme weather hazards has been increasing over the past 50 years.
- ☑ It is now possible, through extreme event attribution methods, to make quantitative statements about the influence of human-induced global warming on specific individual extreme weather events.
- ☑ Extreme event attribution is a formal use of causal inference techniques. Confidence in resulting attribution statements is enhanced when multiple methods, mathematical models, and data sources lead to similar conclusions. These conclusions can be powerful tools for adapting behaviors and infrastructure to climate change.
- ☑ Causal inference together with an understanding of the key physical processes has revealed that the human influence on the climate system, including extreme weather, is indisputable.

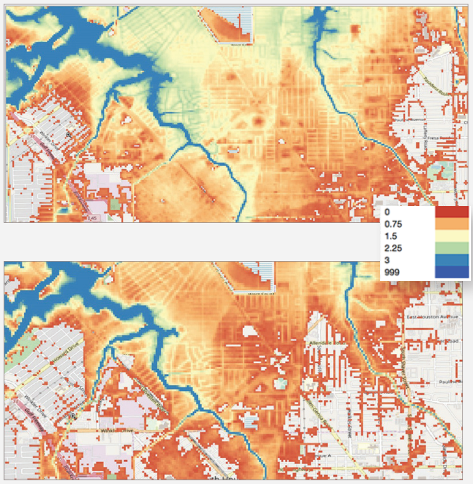
MORE PRECISE RESULTS

Using statistical causal inference, attribution scientists ask two related questions: "Did climate change affect the probability of the occurrence of an event of the observed magnitude?" and "Did climate change affect the magnitude of events of a rarity similar to that observed?" Two classes of causal inference methods can be used to answer these questions.

Pearl causality requires an *i*nterference, often a placebo in clinical medical trials. As there is only one Earth, attribution scientists use climate models instead. Simulations of the actual climate, configured as realistically as possible, are compared to equivalent counterfactual simulations where human contributions to greenhouse gases are removed.

A second class of causal inference statements without interference can be made from the observed data alone. Granger causality is demonstrated by constructing statistical models that utilize parameters to represent natural and human influences. Significance testing of these parameters can reveal their relative roles in the frequency and magnitude of extreme weather events. However, as simple correlation alone does not imply causality, there must be a sound physical mechanism to reinforce any conclusions.

CASE STUDY: HURRICANE HARVEY IN HOUSTON, TEXAS



Event attribution studies of severe storms are a recent development. The first tropical cyclone attribution statements were made in 2017 about Hurricane Harvey, a devastating Category 4 hurricane that made landfall on the United States' Texas coast. The storm dumped a year's worth of rain in less than a week on the city of Houston, where one out of every 4.7 residents lives in poverty.

Three independent groups concluded that global warming significantly increased the storm's precipitation and subsequent flooding. Before these studies, most specialists, including this author, felt that the human-induced increases in extreme precipitation would be dictated by thermodynamics and constrained to roughly 7% per degree Centigrade warming, as dictated by the 19th century Clausius-Clapeyron relationship. However, analysis of Harvey and other hurricanes finds that extreme tropical cyclone precipitation increases with warming at a rate substantially greater than thermodynamics alone, revealing that dynamical processes are also important.

Top: Simulation of the depth of the actual Hurricane Harvey flood in the South Houston neighborhood. **Bottom:** Simulation of the counterfactual Hurricane Harvey flood without the current amount of climate change.

This research was supported by the Director, Office of Science, Office of Biological and Environmental Research of the U.S. Department of Energy under Contract No. DE340AC02-05CH11231 under funding from the Regional and Global Modeling Analysis program.

For the deadly 2003 European heat wave, a group of scientists in the United Kingdom found that such extreme heats were twice as likely with climate change than without, answering the first attribution question. Now, nearly 20 years later, unabated global warming has increased the risk of such heat waves by a factor of ten or more. For the second attribution question, it's estimated that climate change increased the temperature of such rare heat waves from about 0.5°C in 2003 to about 2°C in 2021.

The degree to which global warming alters the risk of a particular heat wave varies. For instance, the 2015 heat wave in Pakistan was both hot and very humid. In high humidity, the air is saturated with water vapor, limiting the evaporative cooling effect of sweating. Attribution scientists concluded that climate change increased the chances of that event by at least a factor of 1,000. It's fair to say that all recent and future extreme heat waves are or will be made more severe by the current levels of climate change.

Extension of statistical causal inference to storms, droughts, and other extreme weather has provided similar information about the role of climate change in these hazards.

COMMUNICATING THE RISKS

Results of attribution studies can provide persuasive evidence of the urgency — and the catastrophic consequences — of climate change and inform decisions about managing risk and selecting adaptation strategies. For example, flood maps that fail to account for climate change are likely to underestimate the actual flood risk. Similarly, the risk to property and life from increased heat wave temperatures and hurricane intensities can be quantified by event attribution.

But as for any causal inference statement, it is important to realize the underlying statistical framing of extreme weather event attribution. Hence, blaming climate change for damage from an extreme event can be correct in a probabilistic sense. Attribution studies also help climate scientists improve their understanding of the physical mechanism of changes in extreme weather as the climate warms. This second purpose is reflected in the IPCC 6th Assessment Report "It is indisputable that human activities are causing climate change, making extreme climate events, including heat waves, heavy rainfall, and droughts, more frequent and severe."

CONCLUSIONS

As the climate continues to warm, the human influence on the severity and risk of extreme weather becomes larger, and attribution statements about extreme weather events are useful for developing strategies to help control vulnerability and exposure and better prevent disasters.

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PINPOINTING THE INDIAN MONSOON

IMPROVING PREDICTABILITY OF THE ANNUAL SUMMER RAINS

The Indian summer monsoon is a dramatic multi-scale phenomenon that profoundly affects the livelihoods of billions of people in the region. Each year, India eagerly awaits forecasts of the monsoon's arrival, as well as its predicted intensity and duration. Not only do the crop-nourishing rains have grave implications for the country's economy, but this annual spectacle also brings joy and relief as the dry summer heat gives way to cooler temperatures and plentiful water. Understanding the dynamics and variations of this complex weather system is critical to improving the predictive capabilities of monsoon forecasts and weather and numerical climate models more broadly.

KEY MESSAGES

- ☑ Mathematical modeling is fundamental to forecasting annual monsoon arrival, intensity, and duration. Such predictions are vital to Indian society, agriculture, tourism, and economic development.
- ☑ Improved modeling capability is key to improving monsoon predictability at different time scales: Daily, monthly, and multi-decadal. Improving models also allows for understanding spatial variations in and future projections of monsoon precipitation.
- ☑ Coupling climate change with models is essential to understanding future monsoon evolution.
- ☑ New data, with tools to analyze these data, are needed to build sufficiently precise models of the intricacies of complex monsoon dynamics.

FROM DRY TO WET

The onset of India's summer monsoon follows solar heating of the Earth's surface in spring and summer. Differential heating leads to warmer lands and cooler ocean waters, producing a temperature gradient that causes a large-scale reversal in the winds over the northern Indian Ocean. The southwesterly winds bring massive cloud bands from the equatorial ocean to the continent and shift the country from a no rain state to a rainy state.

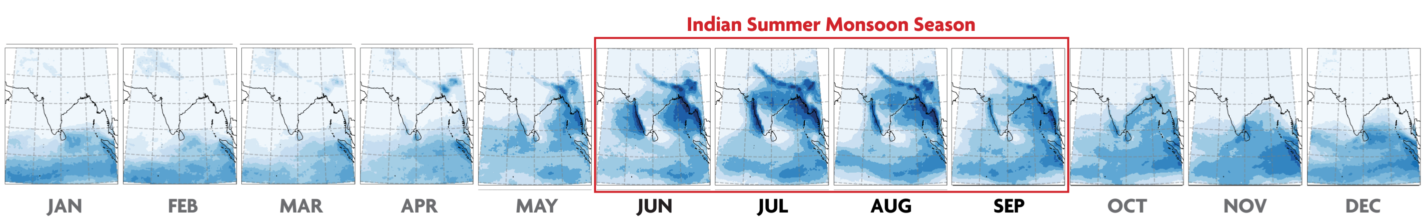
Monsoon rains typically start in June and, by September, will have dumped approximately 850 mm of water and nearly 80 percent of the country's total rainfall. This phenomenon happens with remarkable regularity, both in timing – within 5 to 7 days on either end – and total rainfall, with variations of roughly 10 percent of the average. But these seemingly small variations can have momentous implications for agriculture and the economy.

At a time when one part of India is experiencing severe flooding, the other is likely facing a severe shortage of water. And these circumstances can flip within a few days. As the vast majority of the continent's agricultural production is rain-fed, knowing about any variation in the intensity and duration of the monsoon and when it will start is vital to Indian society, particularly farmers.

WHERE, WHEN, AND HOW MUCH?

A detailed study of monsoon spatio-temporal variations requires a variety of mathematical models, as no single model can capture all aspects of this complex phenomenon. Statistical tools help to extract persistent and interesting patterns of rainfall. Some of the simplest models for capturing the dynamics of such patterns are based on ordinary differential equations. The complex interplay between the rainfall, land, ocean, snow cover, and atmosphere, including greenhouse gases and aerosols, can only be captured by models that require large supercomputers to run.

MONTHLY RAINFALL ON THE INDIAN SUBCONTINENT

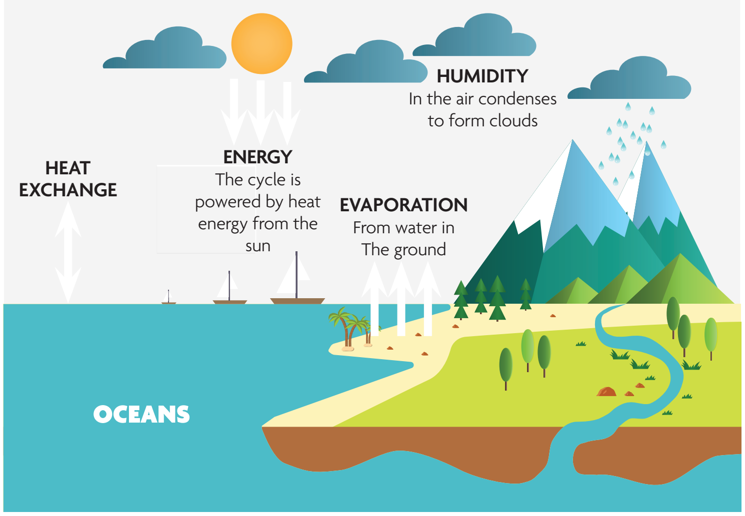


EXAMPLE: MATHEMATICALLY MODELING THE WATER CYCLE

Using mathematical representations of the atmosphere, ocean, and land, models are able to simulate complex weather and climate phenomena such as monsoons and El Niño. The vital role that the Indian summer monsoon plays in the global hydrological cycle is reflected in mathematical models that simulate the dynamics of energy and moisture exchanges within the earth system.

Water evaporates, but the rate of evaporation depends on temperature, humidity, and winds. When atmospheric air is saturated, water vapor condenses to form small water droplets or snowflakes that form clouds. If the droplets grow sufficiently large, they fall to the ground as rain or snow. This water eventually evaporates and the cycle continues.

A simple mathematical model for such a process would take into account the water content of the ground, humidity in the air, and the energy required for this cycle. The ordinary differential equations for these variables are aimed at capturing various processes from simple periodic variations to more unpredictable, chaotic variations. A more complex model would have to include the effects of winds, the oceans, and their interactions with the atmosphere.



Apart from delivering a better understanding of monsoons, mathematical models are also used to make predictions at various time scales. Such predictions can improve decision-making and planning, reduce risks and maximize benefits across various sectors, from agriculture to tourism. For example, accurate predictions of onset or termination dates can help farmers fine-tune the best time to sow, irrigate, apply fertilizer, and harvest their crops.

CONCLUSIONS

In a rapidly changing climate, mathematical models offer a way to assess the effects of climatic changes. While substantial progress has been made in understanding monsoons, the fundamental processes related to clouds, aerosols, and their interactions continue to be exciting but crucial scientific and mathematical challenges. The effect that climate change will have on monsoons remains an active area of research, looking at questions such as 'Will monsoon rainfall decrease or increase?' or 'Will the frequency of intense thunderstorms increase?'.

Beyond the scientific challenge, delivering solutions presents an opportunity to collaborate at a global scale to collect more accurate and relevant data and develop mathematical tools to analyze that data. Collaboration and communication between scientists, mathematicians, policymakers, and society will lead to innovative approaches to mitigate the effects of these uncertain changes, positively impacting the lives of billions of people. Mathematical models can serve as a bedrock for advancing these discussions and improving decision-making.

The Indian summer monsoon is the most prominent of a global monsoon system that also includes the Asian-Australian, American, and African monsoons. More than half of the world's population — most in developing countries — live in monsoon-dominated climates.

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SUSTAINING FISHERIES

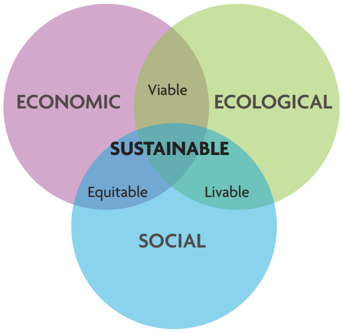
BIOECONOMIC MODELS FOR FISHERIES MANAGEMENT

For nearly a century, mathematics has played a central role in managing the world's fisheries. Mathematical models allow fisheries managers and decision-makers to assess fish stocks and devise management plans that optimize fisheries for maximum gains while preserving the resource. Traditional models, many still widely used today, are simple, static, and often single species-focused. Newer integrated modeling approaches recognize fisheries as complex socio-ecological systems with multiple stakeholders. These dynamic models capture the economic, social, and ecological drivers of fisheries and promise improved support for sustainable fisheries management and decision-making.

KEY MESSAGES

- ☑ Fisheries management must be based on tools that enable managers and decision-makers to harvest for ecological, economic and social sustainability. By coupling economic and ecological dynamics, mathematical modeling makes it possible to develop such tools.
- ☑ Scientists, economists, and decision-makers rely on mathematical models to infer fish stocks from limited data and to devise management plans that optimize fisheries, allowing for maximum gains while preserving the resource. Mathematical models also provide decision-makers with quantitative approaches for evaluating the consequences of alternative actions.
- ☑ Ecosystems are complex and unpredictable, and modeling them will require increasingly sophisticated mathematical theories.

More than a third of the world's fish stocks are being overfished. Overfishing reduces stocks at a rate that the species cannot sustain itself, endangering species and robbing the more than 800 million people who depend on fish and seafood for their food and income. While many — primarily developed — countries are improving the management of their fisheries, progress is needed to make all fisheries sustainable and keep currently sustainable fisheries from becoming unsustainable.

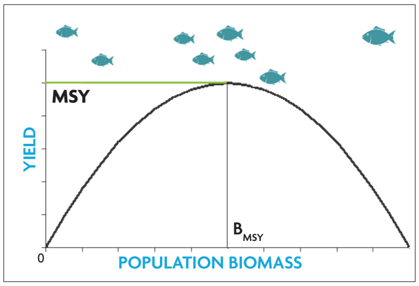


Much has been written about the sustainability of fisheries. While most favor a goal of sustainability, few agree on a universal standard for it. Early approaches that called for fishing less failed to address all desired outcomes: ecological, economic, and social.

Mathematical models have allowed a better understanding of the dynamics of fisheries systems on an ecosystem scale. By coupling ecological and economic dynamics, mathematical modeling makes it possible to develop tools that enable managers and decision-makers to harvest these resources sustainably.

MAXIMUM SUSTAINABLE YIELD

The optimal fisheries management strategy is often described as the strategy that results in the maximum sustained yield (MSY). It's the largest annual harvest that a fish stock can produce and sustain itself indefinitely. MSY can be estimated using surplus production models, first introduced in 1954 by biologist M.B. Schaeffer.



The use of MSY as the objective of fisheries management has been questioned and criticized by scientists and economists alike. Surplus production models are seen as too simple to fully capture the dynamics of populations that are subject to variable recruitment and catchability, interactions with other species, and environmental conditions. Harvesting at MSY is also potentially unstable, as a small decline in population can lead to a positive feedback loop and rapid extinction if harvesting is not reduced.

Economist H. Scott Gordon generalized the Schaefer model to solve for maximum economic yield, the level of harvest that gives the largest net economic profits while reducing the risk of overexploitation. Gordon was among the first to illustrate how an unregulated or open access fishery could lead to economic overfishing.

The Gordon-Schaeffer model is still extensively used to design fisheries management policies, mainly because of its simplicity and applicability in data-poor stocks. But like other static models, it's merely a snapshot of nature, treating the environment as unvarying and ignoring the fact that fish populations experience natural fluctuations in abundance. When dealing with natural populations, a dynamic model is generally preferable.

BIOECONOMIC MODELS

In the 1970s, mathematician Colin Clark was among the first to combine mathematical techniques with insights from capital theory to improve fisheries economics. Recognizing flaws in the existing static models of resource management, Clark devised dynamic models of fishing behavior and harvesting rights that bridged the disciplines of economics and biology and addressed complex issues including discounting, optimal paths toward optimal resource exploitation, and non-malleability of fishing capacity.

Clark and Lamberson applied this mathematical bioeconomic model to the mid-century Antarctic whaling industry. Antarctic whales were heavily harvested beginning in 1925. A rapid buildup of whaling fleets was quickly followed by an equally rapid decline in whale numbers. The blue whale was the first to be depleted below its maximum sustainable yield. Population levels declined steadily until the species was protected worldwide in 1966. By analyzing the projected consequences if a single operator had monopolistic control of the whale fishery, their model revealed the capital and investment parameters that drove whalers to be more interested in short-term profit than long-term sustainability.

VIABILITY THEORY

Scientists have too often failed to recognize and mitigate the harmful social and economic consequences of fisheries management policies. These consequences can be significant and usually last longer than any fishing limitations.

"Viability theory" was first developed in mathematics and later applied to the sustainable management of renewable resources by Jean-Pierre Aubin. It can be used to assist decision-makers in defining and selecting long-term fisheries objectives at an ecosystem level. In contrast to previously described approaches, the objective is not to deliver a single optimal strategy or solution but to ensure viable policies that keep the dynamic system within a constrained set of acceptable states. The viability approach requires fisheries managers and planners to define, upfront, the long-term objectives of the fishery, and the set of ecological, economic, and social constraints to be avoided.

This set of bioeconomic states and decisions, known as the viability kernel, can inform a variety of different policies that respect the constraints. Therefore, viability theory offers more possibilities for negotiation and discussions among stakeholders than the techniques that propose a single optimal policy.

Using the viability approach, economist Vincent Martinet and his team analyzed the recovery paths from the historical crisis of 1994 in the Bay of Biscay *Nephrops* prawn fishery. They compared simulated recovery paths with the estimated historical path. They demonstrated that a less stringent reduction in fleet size would have resulted in a more rapid recovery towards a viable fishery.

Laayoune Harbor, Morocco. Maritime fishing accounts for 2 to 3 percent of Morocco's GDP, and employs an estimated 600,000 people. Here, a fishing moratorium would be economically and culturally disastrous. © Najib Charouki

CONCLUSIONS

Continued use of mathematical models in fisheries management requires a critical appraisal of the different model forms, with all of the advantages and disadvantages of each. Despite advances in modeling, available models have only had a limited ability to inform management and decision-making, integrating socio-ecological dynamics. There is a need to build upon existing models, incorporate new techniques, and extend the ability of these approaches to capture the ecological, economic, and social drivers of fisheries and improve their sustainability.

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MEASURING BIODIVERSITY

A DATA-DRIVEN APPROACH TO GLOBAL NATURE CONSERVATION

Global biodiversity is in crisis. Modern species are going extinct at a rate hundreds of times faster than they have at any other point in human history. The post-2020 biodiversity framework guides global action to preserve and protect nature and its essential services. Producing indicators to track progress on the targets will require monitoring systems to produce primary data and the classification and analysis of that data. Mathematics underpins all of these actions, from the indices used to measure biodiversity to the algorithms speeding up classification. Mathematical and scientific cooperation and technology transfer will help ensure 2030 targets are met en route to the 2050 goal of 'living in harmony with nature'.

Healthy ecosystems are the foundation of human health and well-being. Extraordinarily high extinction rates have degraded ecosystem structure and functionality, lending urgency to global commitments to mitigate biodiversity loss. Numerous efforts have been made to objectively measure biodiversity. Typically this is done using biodiversity indices.

KEY MESSAGES

- ☑ Biodiversity is critical to maintaining healthy ecosystems. Biodiversity indices allow scientists to quantitatively estimate biodiversity from field observations.
- ☑ Biodiversity indices allow ecologists to estimate biological variability in space and time, set biodiversity goals and measure progress toward them, and design interventions to enhance biodiversity and ecosystem sustainability.
- ☑ Non-biased and accurate data is required to compute biodiversity indices. Traditionally, data are collected through ecological field sampling techniques. In recent years, crowdsourcing, citizens science, and artificial intelligence are increasingly used to support both sampling and data analysis.
- ☑ The mathematical underpinnings of biodiversity indices vary widely. Mathematical axioms can be used to choose the best index given the context of a particular application or ecological study.

DOES MORE MEAN BETTER?

The most iconic and simplest measure of biodiversity is species richness — the absolute number of species in an ecosystem. Although it's a key metric widely used by many conservation planners, species richness as a quantitative estimate of biodiversity has shortcomings. Most notably, measurements of species richness tell very little about how common or widespread the individual species are. All species are treated as equal, from the extremely rare to the incredibly abundant. Invasive species, for example, might add to the immediate richness, but their continued presence might ultimately decrease richness as endemic species are squeezed out. Richness is also particularly sensitive to sampling effort, such that the number of species rises with the number and size of sampling units. Measures must be taken to standardize samples and limit sampling errors. Mathematical tools have been developed to optimize sampling design to maximize information and minimize effort.

EVEN OR UNEVEN?

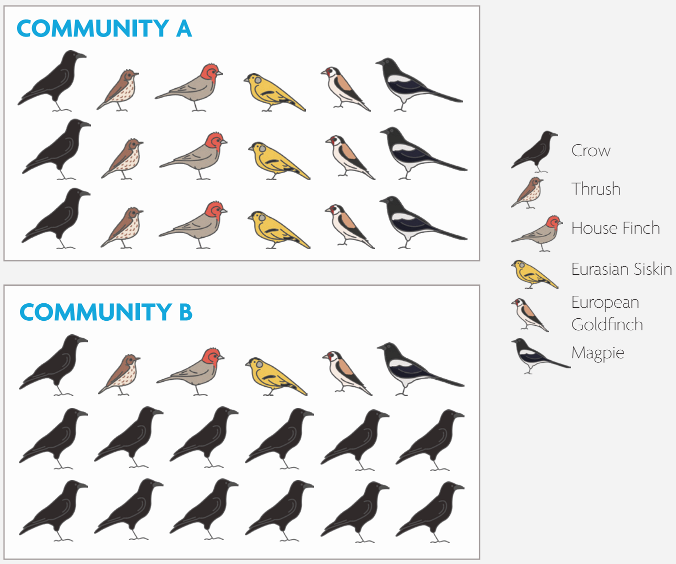
Another important component of ecological diversity is how evenly the individuals are distributed among different species in the community, referred to as evenness. Studies suggest that uneven communities are less resilient to shocks and stresses and more susceptible to invasions.

The most widely used evenness index in ecological literature is based on information theory, namely Claude Shannon's early work on entropy. The Shannon-Wiener Diversity Index quantifies the uncertainty in predicting the species of an individual taken at random from a sample. This uncertainty is largest when the number of individuals in each species is the same. Shannon's Index is generally more influenced by the number of rare species in a community.

One of the best known and earliest evenness measures is the Simpson's Index, used for large sampled communities. This index measures both the number of species present and the proportion of each species. It expresses the probability that two individuals drawn at random belong to the same species. A large value implies a clumping of individuals in a few species, and a small value suggests a more uniform distribution of individuals among the species. Simpson's Index is particularly sensitive to changes in the relative abundances of the most important species.

One problem with traditional diversity measures, in particular evenness indices, is that not all species are equal

VISUALIZING ECOLOGICAL DIVERSITY



In this example, ecologists are comparing passerine birds in two European temperate broadleaf communities. Both Community A and Community B have 18 individuals of different species. In both communities, richness is equal to six. Evenness is how evenly the species are distributed. In Community A, all species are present in equal abundances. Community B is very uneven as it's dominated by crows.

— functionally, evolutionarily, or ecologically. As species' functional and ecological traits result from evolution, some ecologists have suggested incorporating measures of phylogenetic or taxonomic diversity. Evolutionary diversity metrics can help decision-makers assess conservation values of different areas and prioritize conservation of regions that are more functionally and genetically diverse and thus will have the most options to respond to future change.

CHOOSING THE BEST INDEX

A vast number and variety of indices are actively used to measure ecological diversity. The mathematical underpinnings of each of these indices vary widely, making it difficult to choose the best one given the context of a particular application or ecological study. Some researchers have turned to mathematical axioms — unprovable principles accepted as true — to serve as a premise or starting point. Axioms are used to identify the most important properties of diversity indices and select them based on which axioms they satisfy or fail to satisfy. Axiomatic approaches have been widely used in other areas. The famous Arrow Axioms, for example, illustrate the challenges of creating a fair voting structure.

AI MEETS CITIZEN SCIENCE

All biodiversity measurements require data, but acquiring it through conventional field sampling or even remote sensing can be time-consuming and costly. Crowdsourcing and citizen science are proving to be increasingly useful approaches for collecting and classifying biometric data. The Snapshot Serengeti project enlisted volunteers to help classify images collected from hundreds of motion-activated camera traps in Tanzania's Serengeti National Park. More than 30,000 volunteers have helped make over half a million image classifications to date.

Introducing artificial intelligence can address many risks associated with citizen science data collection, from observational biases to classification errors. Artificial intelligence and machine learning techniques are now being used to identify and validate classified images. Deep learning saved an estimated 17,000 hours of human effort in the Snapshot Serengeti project.

When animal images are collected using camera traps, individuals of the same species are most commonly captured in the same habitat. This can lead to computer vision systems classifying the background instead of the animal, resulting in bias. Approaches like co-segmentation get around this by automatically isolating the object of interest without any manual input. State-of-the-art object identification systems use bounding boxes, then resample features for the boxes, and finally use machine learning to classify the objects. Mathematical tools are being developed to train neural networks to identify individuals based on biometric data, such as feather, coat, or skin patterns; facial features such as whisker patterns; footprints; and vocalizations.

CONCLUSIONS

Measurements of biodiversity are essential for monitoring the health of ecosystems. New tools are needed to guide, monitor, and measure progress to halt the loss of biodiversity by 2030 and achieve recovery and restoration by 2050. Mathematical approaches can help strengthen the development and selection of indices and indicators, data collection and validation techniques, and the associated methodologies.

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LISTENING IN ON WILDLIFE

ANIMAL SOUND CLASSIFICATION WITH DEEP LEARNING

Recent reports of worldwide biodiversity declines emphasize the importance of monitoring populations and protecting species and their habitats. Traditional surveying methods are costly and often invasive. For vocalizing animals, passive acoustic monitoring offers a more cost-efficient, scalable, and noninvasive means of data collection. But processing the often-massive audio datasets remains labor-intensive and requires significant data storage. As demonstrated by a cross-disciplinary and cross-continental team of researchers, artificial intelligence techniques, in particular deep learning, can provide a rapid, efficient way to process wildlife sound data and ultimately better support conservation efforts.

The critically-endangered Hainan gibbon (*Nomascus hainanus*) is the world's rarest primate and one of its rarest mammals. The last surviving population of just 30 or so individuals lives only in a small remnant forest in the Bawangling National Nature Reserve on China's Hainan Island.

Monitoring this rare animal has traditionally relied on labor-intensive and intermittent field surveys, where teams record the boisterous gibbons from elevated listening stations. Effective and cost-efficient tools are urgently needed to continuously and noninvasively monitor this and other immensely vulnerable populations and possibly help detect previously undiscovered new ones.

A large collaborative team of international researchers set out to develop a new passive acoustic monitoring (PAM) method to automatically detect and isolate Hainan gibbon calls. In 2016, they deployed autonomous sound recorders in Bawangling to passively eavesdrop on the animals over nearly six months. The team captured more than 6,000 hours of audio recordings totaling hundreds of gigabytes of data. Such massive volumes of data — typical of PAM studies — are expensive to store and time-consuming to process and classify manually. Automating the collection and analysis of such data enables longer and cheaper monitoring and ultimately can facilitate a near-real-time understanding of population dynamics and more timely conservation responses.

KEY MESSAGES

- ☑ For vocalizing animals, their distinctive sounds can provide useful information on their distribution, abundance, and behavior. Passive acoustic recorders allow researchers to efficiently and noninvasively monitor species populations and how they're moving or changing over time.
- ☑ Continuous passive acoustic monitoring produces huge datasets that are time-consuming to process and require costly storage. Advancements in artificial intelligence are allowing researchers to create complex mathematical models trained to recognize the calls of a particular species at a fraction of the time of conventional approaches.
- ☑ Automating the processing and classification of acoustic data, not only enables cheaper and longer monitoring, but allows researchers to proceed more quickly from data collection to analysis, generating near-real-time insights on population dynamics, and enabling timely management and conservation decision-making.

VISUALIZING SOUND

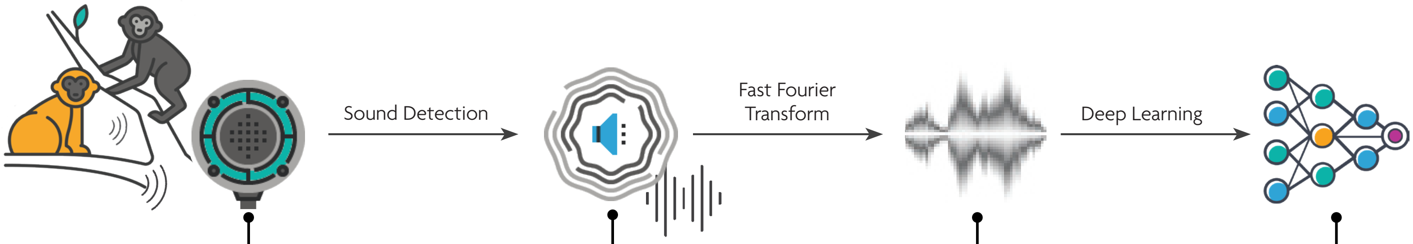
Acoustic sensors detect and convert incoming sound waves into an electrical signal, digitally stored as a sequence of binary numbers. As it's difficult for the human ear – or computers for that matter -- to distinguish sounds in this format, the signal must be processed to recover frequency information and render visually. This processing is commonly done using a mathematical technique known as Fast Fourier Transform, which generates a spectrogram — a time-frequency representation that enables sound to be visually identified.

Calls of specific species are typically detected and classified manually, which is subjective and, because datasets are so large, time-consuming, and labor-intensive. Artificial intelligence technologies can facilitate automated or semi-automated analyses, significantly reducing the time and effort required to detect and classify calls.

MACHINE LISTENING

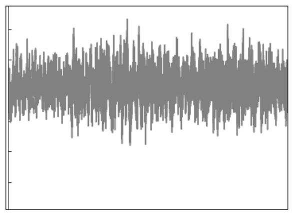
Of the many artificial intelligence techniques, deep learning has proven particularly useful in sound detection and classification. Deep learning models can be trained to pick

AN AUTOMATED CLASSIFIER FOR IDENTIFYING HAINAN GIBBON CALLS



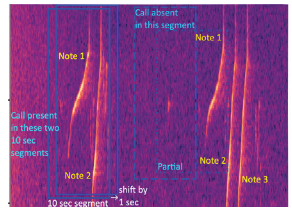
PASSIVE ACOUSTIC RECORDERS
Acoustic recorders noninvasively capture sound data from Hainan gibbons and other species in the Bawangling National Nature Reserve.

(© Tonglei Zhou, Hainan University)



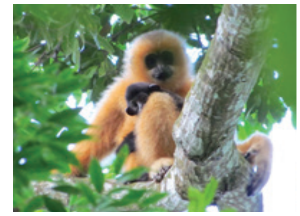
ACOUSTIC DATASETS
The acoustic data is digitally stored as amplitude values, which are recorded at a fixed number of observations per second.

Hainan gibbon call displayed using raw amplitude values



SPECTROGRAM
To classify acoustic data, amplitude values are converted to a spectrogram. This is typically done using a mathematical technique known as Fast Fourier Transform.

Hainan gibbon call displayed as a spectrogram



CLASSIFIED DATA
Using deep learning techniques, researchers develop models to automatically detect and rapidly classify gibbon calls.

A critically-endangered Hainan gibbon. (© Jessica Bryant, Zoological Society of London)

out patterns and abstract concepts in data in a similar way to the human perceptual system.

Researchers can develop and train models to automatically recognize and rapidly classify a specific animal's call using these approaches. For example, the Hainan gibbon classifier model not only generated highly accurate predictions, but it processed more than 6,000 hours of audio recordings in just two days, a fraction of the time it would take to process manually.

FILLING A GAP FOR RARE SPECIES

While artificial intelligence and acoustic monitoring have the potential to transform the field of wildlife management and conservation, some challenges remain, among these background noise sensitivity and reliance on large training datasets. When available, wildlife call libraries are often too small to effectively train artificial intelligence models, limiting their use for rare species or those for which limited call samples exist. Newer algorithmic methods, such as the few-shot learning approach that enables detection with very limited datasets, could prove pivotal in developing models to detect profoundly rare species like the endangered Hainan pheasant peacock for which only three publicly available recordings exist.

CONCLUSIONS

Advancements in artificial intelligence are allowing researchers to create complex mathematical models trained to recognize the calls of a particular species at a fraction of the time of conventional approaches. While the progress has been promising, further research will enable the development of more reliable models. Cross-disciplinary teams made up of ecologists, mathematicians, and data scientists have played a pivotal role in advancements to date and, in the long term, will help decision-makers better manage and conserve endangered species and their habitats.

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BATTLING INVADERS

MATHEMATICAL APPROACHES TO INVASIVE SPECIES MANAGEMENT

Invasive species are a leading threat to the world's natural and human systems. With expanding global trade and climate change, they're increasing worldwide, costing the global economy upwards of $1.4 trillion per year and causing the extinction of hundreds of species. Using mathematical models, scientists can better understand the dynamics of invasive species and predict their expansion and their potential responses to climate change. For decision-makers and managers, mathematical approaches offer tools to identify and prioritize strategies to reduce or stop the spread of invasive species, as well as the ability to quantify costs and allocate resources to control and eradicate damaging populations.

KEY MESSAGES

- ☑ Unchecked, invasive species can devastate ecosystems, changing habitats and starving out native species for food and other resources. Mathematical modeling provides managers and decision-makers with powerful tools to efficiently prioritize and allocate resources for limiting their spread.
- ☑ Mathematical models can help scientists predict the impact of invasive species on threatened and endangered native species and quantify efforts required to control and eradicate damaging invasive populations.
- ☑ A warming atmosphere is likely to allow more species to move further, and for invasive plants, it gives some species a longer growing season. Mathematical and statistical simulations can help predict future risk by forecasting which ecosystems might be invaded and the potential impacts if they are.

Invasive species establish and spread rapidly outside their natural borders, often introduced into new areas — intentionally or inadvertently — by humans. They can severely impact ecosystems by outcompeting or preying on native species and introducing new pathogens. Increasing global trade, climate change, and habitat loss have accelerated their movement. And today, invasive predators, pests, and plants have contributed to hundreds of species extinctions and caused enormous natural and economic damages to agricultural, forest, freshwater, and marine systems.

Controlling or eradicating invasive species is often costly and resource-intensive. To prioritize management actions, scientists mathematically model the effects of invasive species in non-native systems. Usually, this starts with understanding or predicting the competitive or predatory impacts on native species.

MODELING INVASIVE IMPACTS

One of the earliest models in mathematical ecology is the Lotka-Volterra equation, a predator-prey model proposed in 1925 by Alfred Lotka, and American biophysicist and Vito Volterra, an Italian mathematician. Based on differential equations, this simple model forms the basis of many models used today.

Single-species predator-prey models simulate the dynamic interactions between two species. As an example, feral cats are non-native predators known to significantly impact biodiversity, particularly on islands. They've only recently been introduced to isolated islands in the Mediterranean Sea, which were free of predators in the past. The Yelkouan Shearwater, a medium-sized seabird, nests on these islands during their November breeding season. Though agile at sea, the birds are clumsy and vulnerable when nesting on the ground, making them an easy target for introduced predators like feral cats. On the French island of LeLevant, feral cats were responsible for the yearly death of about 800 to 3,000 of these birds. As a result of predation, Yelkouan Shearwater populations are steadily declining, and the species is now listed as vulnerable to extinction by the International Union for Conservation of Nature.

A model of these two species describes the evolution over time of their respective abundance, given key parameters of their population dynamics and interactions:

- **Prey growth rate**. The population grows exponentially when the population is small, and the larger the growth rate, the faster the growth.
- **Prey carrying capacity**. The maximum population size that an area can sustain. Population growth slows as it approaches carrying capacity.
- **Predator-prey interaction term.** Interactions between the predator and prey result in the predator population increasing and the prey population decreasing.

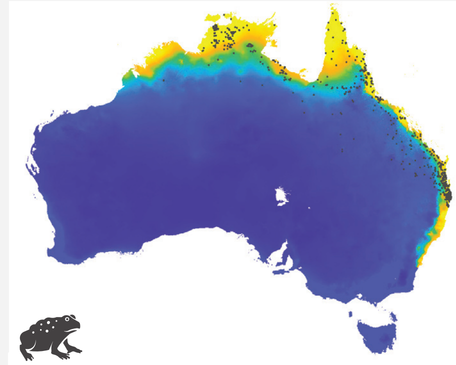
SPECIES DISTRIBUTION MODELS: THE CASE OF AUSTRALIA'S CANE TOAD

Species distribution models (SDMs) are among the most widely used modeling approaches in conservation ecology. They use environmental data such as land cover, precipitation, temperature, elevation, and vegetation indices as predictors of suitable habitats. Without suitable habitats, invasive species cannot establish sustainable populations in new areas. The environmental data needs to be measured in the species' existing habitats and is then applied to estimate habitat suitability in new, unoccupied areas. For non-native species, SDMs can be a useful tool for predicting ranges of invasion, helping to guide monitoring and management efforts, and targeting resources efficiently.

Cane toads are indigenous to the Americas — ranging from Peru to Texas— but have been introduced to many locations worldwide. In Australia, the giant toads were intentionally introduced in 1935 in a misguided and altogether unsuccessful effort to control cane beetles that were destroying sugarcane plantations. With an incredible reproduction cycle and no natural predators in part due to their toxic skin secretions, the toad populations exploded in Australia, and they now occupy more than 1.2 million square kilometers.

Scientists can run statistical models on the current distribution of cane toads and their native distribution in South America, to better understand the types of environments they can persist in. The results reveal that the species will continue to invade both south and west Australia in the years to come.

As with any model, the utility of SDMs depends in large part on the type and quality of data. SDMs do not typically account for biotic interactions or dispersal limitations, two variables that are known to constrain species abundance. So if a species cannot occupy suitable habitat because it can't reach it or another species is blocking it, SDMs might not predict that the habitat is suitable. Likewise, climate change has pushed species to occupy sub-optimal habitats because their existing habitat has changed, but they haven't been able to move fast enough to new areas.



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- **Population control term**. Through different removal actions, predator populations can be suppressed - or even eradicated - to allow the prey population to recover.

Each of these parameters can vary over time. They may also vary with climate change, as areas that were once unsuitable for invasion become more hospitable due to changes in temperature, rainfall, and other environmental parameters. Using mathematical models, scientists can predict how these changes will affect invasive species management over time.

Predator-prey models form the basis of many mathematical tools that support management and policy decisions. In practice, it's often challenging to estimate these parameters precisely. To realistically model a proposed management strategy, the cost of removing each individual of the invading species must be estimated and included. Invasive species management can be improved by adopting an adaptive management approach, whereby models are used to guide initial control and eradication actions. Results of these early interventions are then used to understand system dynamics and adjust and improve future management actions.

CONCLUSIONS

Data collection is important in constructing and validating a mathematical model. And for invasive species, in particular, this data must be collected in conjunction with species management actions. By first consulting with mathematicians, scientists can ensure that any data collected can fit within analysis workflows and realize maximum value from data collection efforts.

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PRESERVING PRIVACY

THE POWER OF MANY WITH COLLABORATIVE MACHINE LEARNING

It's challenging to build accurate machine learning models in domains where data governance and privacy prevent sharing of data. Recent advances in federated learning enable building complex machine learning models trained in a distributed fashion such that private data never leaves a given participant or institution. A higher-quality model can be trained by leveraging a more extensive and more diverse dataset beyond what could be done with the datasets of any one party. And because all parties involved keep their sensitive data local and private, the approach is particularly appropriate for industries where data protection regulations impose barriers to data sharing, such as healthcare, banking, and security.

KEY MESSAGES

- ☑ Federated learning trains machine learning models on separate datasets that are distributed across different parties. Instead of aggregating their data, participating institutions train on the same model, using their own private, locally-stored data. Model parameters are pooled to a central server, which aggregates the contributions to a new, composite model.
- ☑ By enabling multiple parties to train collaboratively without the need to exchange or centralize private data sets, federated learning can achieve the full learning capacity of the data and facilitate large-scale multi-institutional collaborations, while overcoming technical, competitive, and data ownership concerns.
- ☑ In medicine and banking, federated learning helps to meet the requirements of data protection regulations such as the European General Data Protection Regulation (GDPR)24 and the United States' Health Insurance Portability and Accountability Act (HIPAA).

Machine learning using deep neural networks has been successfully applied across a wide range of problems, enabling solutions heretofore unimaginable even a short time ago. In the past five years alone, deep neural networks have allowed for automatically recognizing objects of interest in images, responding automatically to text-based queries, and automating the simultaneous translation and summarization of audio. This remarkable progress has been achieved through a combination of increased computational power, the development of superior algorithms, and the vast quantity of data available to train these data-hungry models.

However, such vast quantities of data may not be readily available to all those who wish to train deep neural networks. New applications of deep learning have been hindered precisely by a lack of adequately large and diverse training datasets. In some cases, a considerable amount of data exists, but it cannot be centrally collected for legal or regulatory reasons. The field of federated learning has emerged to address this impediment and allow deep learning to bring value to a broader set of applications.

Conventional machine learning uses training data collected, often from many different sources and aggregated on a single machine. However, this centralized training approach can be privacy-intrusive if the data contains personal identifying information. In federated learning, machine learning models are trained by multiple parties together in a distributed fashion, whereby each party can keep its private data on its local system. Model parameters — not the model data — are sent periodically to a central server and aggregated to a new, composite algorithm, then disseminated back to the parties for use in local model training.

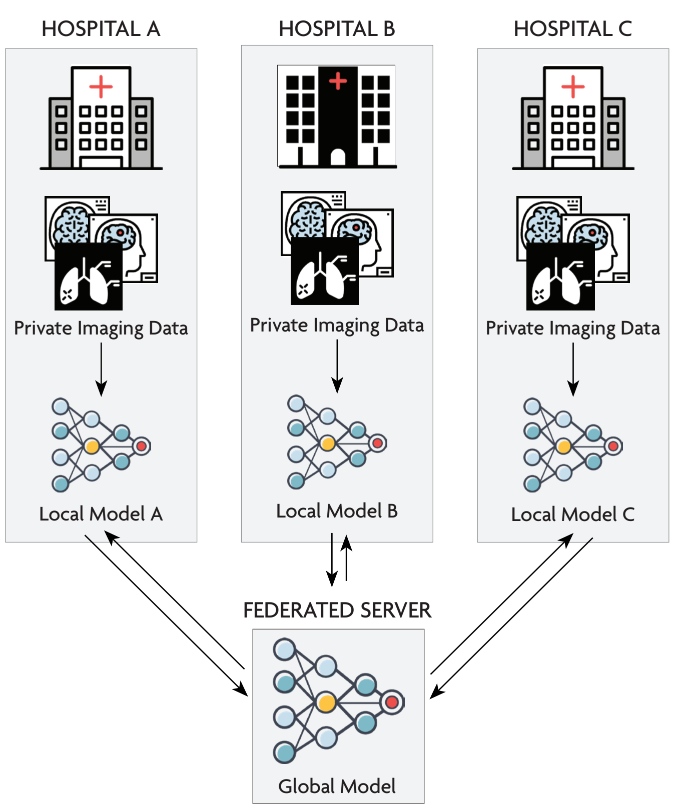
In this way, each participant's model can be improved by others' data, without requiring any transfer of the data itself. The result is that each party involved in the federation can develop a machine learning model for their application using their own data that leverages the privacy-preserved data from other parties to obtain better, more accurate models than they could alone. This federated approach allows bringing the power of many to the few without compromising the integrity of the data that belongs to each participant.

ENHANCING DISEASE DETECTION TOOLS

Machine learning models have shown tremendous potential in disease prediction, but they require large, diverse datasets to train the algorithms. That's often hard to achieve in practice. For example, while a single hospital has medical imaging that's been evaluated for disease pathology, the number of these scans is likely not large or diverse enough to effectively train and validate large-capacity neural networks. This can be mitigated by training with data from multiple institutions, but privacy regulations make it very difficult to pool patient scans with other institutions. However, under appropriate safeguards, hospitals can participate in federated learning methods.

Each participating hospital would train its local disease prediction model and send only the model parameters to a central server without exposing private patient information or transferring images. The resulting federated learning model could therefore be trained on a dataset many times the size of those available to each of the hospital sites, thereby providing significantly higher accuracy and generalizability, reducing institutional biases, and enabling transferability to new disease types, possibly types not seen in the hospital's own patient pool. By personalizing the federated learning-trained models, each hospital gains the best of both worlds: A disease prediction model validated on its own patient population while being able to recognize diseases only seen in other hospitals.

FEDERATED LEARNING FOR MEDICAL IMAGING



IDENTIFYING AND DETERRING FINANCIAL FRAUD

Financial fraud costs the banking industry billions of dollars annually. And while there is a significant appetite for automating systems to recognize signs of financial fraud, several challenges hinder the development of accurate and efficient fraud detection systems. Financial datasets are highly unbalanced, meaning there are significantly fewer samples of fraudulent transactions than legitimate ones. Data privacy and other regulations also impose barriers to sharing financial transaction datasets across banks, institutions, and country borders. As a result, it's difficult to accurately train models to recognize fraud patterns and detect illicit activity.

While a single bank may not have enough confirmed fraud cases to train models effectively, many banks joining together likely would. Using federated learning methods, financial institutions can collectively benefit from a shared model, which has seen more fraud than any single bank alone, without sharing their client's private data with each other.

CONCLUSIONS

Deep neural networks have transformed the ability to use and benefit from large datasets in many applications. Still, they have been slow to demonstrate the same value in domains where privacy or competitiveness prevents data sharing. Federated learning has emerged to allow privacy-preserving distributed machine learning solutions that overcome current shortcomings and accommodate data privacy regulations in healthcare, banking, and security. Governments can help drive such advances by facilitating the creation of federated learning processes and standards across key industries and support research to expand the problems that can be solved with federated learning.

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FINDING THE MISSING

COMPLEX NETWORKS ENABLE SEARCHES FOR MISSING PEOPLE

16 PEACE, JUSTICE AND STRONG INSTITUTIONS

Across the world, hundreds of thousands of people go missing in connection with armed conflicts, other situations of violence, migration, and mass disasters. These disappearances cause incalculable suffering for their families and communities and are an obstacle to peace. Policymakers need better data and tools to objectively address disappearances and advance humanitarian objectives. Complex network analysis can be a powerful instrument for searching for and collecting relevant information on the missing in multiple contexts. Moreover, such approaches have the potential to highlight information that otherwise would not be evident.

International humanitarian and human rights laws contain provisions to ensure the "dead are managed in a proper and dignified manner and to clarify the fate and whereabouts of missing persons." A lack of reliable statistics on the number of missing persons as well as decentralized systems to address the problem challenge the design and implementation of policies to address disappearances. In some cases, the mathematics of complex networks combined with statistical techniques can assist the search process by making it possible to exploit clues to identify clusters of people that may share the same fate.

THE INVISIBLE LINKS IN NETWORKS

A complex network is a set of connected nodes that interact in different ways. Systems that take the form of a network are common and wide-ranging in the world, from food webs to postal delivery routes, high-voltage transmission networks, and social networks of friends or other connections of individuals, business relationships, or organizations. Studies suggest that properties of complex networks — notably, community structure and hierarchical organization — can help explain the behavior of the underlying systems. For example, groups of strongly connected nodes often correspond to known functional units on the system.

In some cases, networks contain a lack of information, both about nodes and links. In recent years, scientists have developed mathematical techniques to predict missing network connections in real-world systems.

KEY MESSAGES

- ☑ A complex network consists of nodes connected by links. In social networks, nodes represent individuals and the connecting links are the relationships between the respective individuals.
- ☑ Complex networks provides a powerful tool to help clarify the fate of missing people by classifying individuals sharing common properties or other similarities within groups or clusters.
- ☑ The structure of a complex network helps researchers to suggest hypotheses that can be explored later using other information and also tested statistically.
- ☑ The structure of a complex network can be further honed to refine results. This could be done, for instance, by classifying the connections into strong or weak connections, or by assigning a number to represent connection strength.

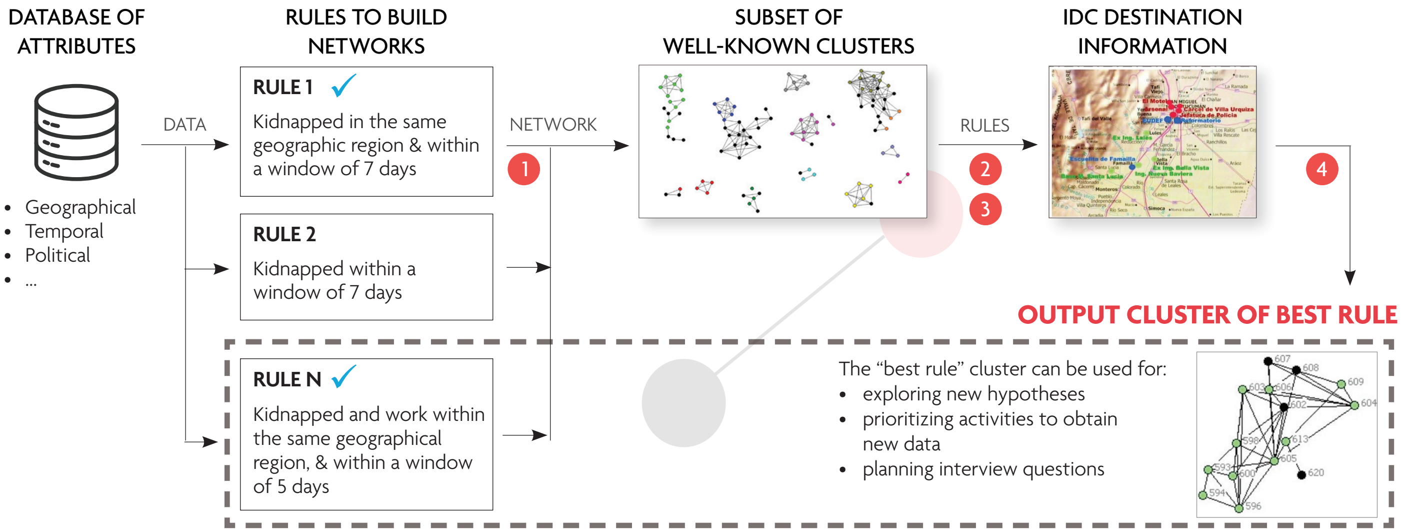
EXPLOITING CLUES

In 2011, scientists with the Argentine Forensic Anthropology Team and Conicet set out about building a complex network reflecting people who disappeared during the country's military dictatorship that was in power from 1976 to 1983. Nodes represent individuals, and links between the nodes represent explicit or implicit relationships between the individuals at some point. For example, the individuals may have lived in the same area, belonged to the same political group, or disappeared inside an interval of 7 days. These relationships could vary over time.

Clusters of interconnected nodes emerged within the network. The nodes were classified based on how much information was known of the individual's fate. To build the network, the team first used anthropological and forensic information to classify 64 nodes into 12 color-coded reference groups. These nodes connect with 41 other nodes, colored black. Considering this subset of 105 nodes, quite a bit was known about 64 of them, including their fates, and less was known about the destinies of the remaining 41 nodes.

The team's goal was to determine a structure of clusters in the complex network compatible with these reference

CONNECTING THE DOTS TO HELP LOCATE ARGENTINA'S DISAPPEARED



1 Database attributes are used to build networks using different rules.

2 A set of well-known clusters, called reference groups, is used to define "compatible rules" (✓).

3 Information theory is used to define two magnitudes that help quantify the rules to best recognize the reference groups.

4 A hypothesis test is used to identify the "best" of the compatible rules, using Illegal Detention Center (IDC) information on a subset of cases.

groups. Black nodes in a cluster of colored nodes might suggest the individuals share the same fate. For example, they may have been held captive in the same illegal detention center. It also suggests what other information needs to be collected, who should be interviewed, and what questions need to be asked.

There are many different ways to choose relationships between individuals in these analyses, and different relationships may lead to different networks. Determining which one to choose involves testing different types of relationships to find a set of compatible rules that best discriminates the reference groups. To select the set of compatible rules that generates the best clusters, the team used Illegal Detention Center (IDC) destination data in combination with statistical inference techniques.

The method was also applied to investigate the circuit of Illegal Detention Centers, where several thousand people were held captive during the Argentinian dictatorship. Two criteria were used for relationships. Strong relationships were those between individuals with the same political affiliation and kidnapped in the same region within a window of 7 days. Weak relationships were those between individuals of the same region, which disappeared in a window of 3 days.

These techniques can be used globally to analyze and connect information concerning the inferred fate of other missing people. Relationships between travel companions along African or Central American migratory routes, for example, might shed light on their whereabouts and assist search and information-collecting activities. In migration contexts, it could be particularly helpful to work with networks at different locations and times, for example, before the departure, during the trip, and upon arrival to the final destination. Nodes or links may change over time, and the corresponding clusters associated with a given node may suggest a set of clues that, exploited together, reveal better information.

CONCLUSIONS

Complex networks have been studied across a range of topics, from genetics to scalable communication networks, vaccination strategies, and ecosystem stability and function. Sophisticated mathematical techniques, combined with computational tools, can help understand and exploit a complex network's very rich structure and dynamics to help answer fundamental questions that challenge humankind.

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Mathematics for Action: Supporting Science-Based Decision-Making focuses on engaging stories of mathematics in action. Written by mathematicians and thought leaders from across the globe, the collection of briefs provides a fascinating demonstration of the role of maths in addressing the world's most pressing challenges in the face of accelerating global change. They cover a wide spectrum of topics related to the Sustainable Development Goals, from drawing maps of poverty to measuring the gender gap, modeling a pandemic and food systems, forecasting climate change, and measuring biodiversity. These issues are complex and multi-faceted and call for diverse perspectives and interdisciplinary solutions.

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